

Name of course: Core CBCS

Scheme/Mode of Examination: CBCS Semester –III

Name of the Paper: Statistical Methods for Economics

UPC/Subject Code: 12271303

Duration: 3 Hrs.

Maximum Marks: 75

*Instructions:* Answer any four questions. All questions carry equal marks. Answers may be written either in English or in Hindi but the same medium should be used throughout the paper. The use of a simple non-programmable calculator is allowed. Statistical tables are attached for your reference. In all calculations, figures should be rounded to two decimal places.

निर्देश: किन्हीं चार प्रश्नों के उत्तर दें। सभी प्रश्नों के समान अंक हैं। उत्तर अंग्रेजी या हिंदी में लिखे जा सकते हैं लेकिन पूरे पेपर में एक ही माध्यम का उपयोग किया जाना चाहिए। एक साधारण गैर-प्रोग्रामेबल कैलकुलेटर के उपयोग की अनुमति है। सांख्यिकीय टेबल आपके संदर्भ के लिए संलग्न हैं। सभी गणनाओं में, आंकड़ों को दो दशमलव स्थानों पर गोल किया जाना चाहिए।

Q1. i) A diagnostic test for a pancreatic disease yielded the following results, when applied on 10,00,000 individuals.

	Number of individuals that actually have the disease	Number of individuals that do not have the disease
Number of Tests that indicated the presence of the disease	4885	73,630
Number of Tests that indicated the absence of the disease	115	9,21,370

- What is the probability that an individual who does not have the disease tests positive?
- What is the probability that an individual who tests positive actually has the disease?
- Which one of the above probabilities should be lower for the test to be an accurate one?

ii) Assume that the time saved by putting FASTags on cars is normally distributed with true standard deviation 0.75.

- Compute 96% confidence interval for the true average time saved based on 35 cars with a sample average of time saved being 8.57 minutes.
- Will your answer change if time saved by putting FASTags on cars is not normally distributed?
- How large must be the sample size if the width of the 92% interval is to be 0.50.

iii) A person will win an amount of Rs500 in a game of cards, if he draws a Jack, a King, a Queen or an Ace, from a well-shuffled deck of 52 cards. If the amount charged in order to play the game is Rs100, should the game be played?

iv) A box contains 5 red balls and 10 white balls. Another box contains 12 red balls and 13 white balls. A ball is randomly chosen from the second and transferred to the first box. Thereafter, a ball is randomly chosen from the first box. What is the probability that the ball chosen is white?

Q1) i) 10,00,000 व्यक्तियों पर लागू किए जाने पर, अग्नाशय की बीमारी के लिए एक नैदानिक परीक्षण से निम्नलिखित परिणाम प्राप्त हुए।

	उन व्यक्तियों की संख्या जिन्हें वास्तव में यह बीमारी है	उन व्यक्तियों की संख्या जिन्हें रोग नहीं है
टेस्ट की संख्या जिसने रोग की उपस्थिति का संकेत दिया	4885	73,630
टेस्ट की संख्या जो बीमारी की अनुपस्थिति का संकेत देती है	115	9,21,370

क) इस बात की क्या प्रायिकता है कि जिस व्यक्ति को यह रोग नहीं है उसका परीक्षण सकारात्मक होगा?

ख) क्या संभावना है कि सकारात्मक परीक्षण करने वाले व्यक्ति को वास्तव में यह बीमारी है?

ग) परीक्षण के सटीक होने के लिए उपरोक्त में से कौन सी संभावना कम होनी चाहिए?

ii) मान लीजिये कि कारों पर फास्टैग लगाने से बचा हुआ समय सामान्य रूप से सही मानक विचलन 0.75 के साथ वितरित किया जाता है।

- क) 35 कारों के आधार पर बचाए गए वास्तविक औसत समय के लिए 96% विश्वास अंतराल की गणना करें, जिसका नमूना औसत समय 8.57 मिनट बचा है।
- ख) यदि कारों पर फास्टैग लगाने से बचाए गए समय को सामान्य रूप से वितरित नहीं किया जाता है तो क्या आपका उत्तर बदल जाएगा?
- ग) यदि 92% के अंतराल की चौड़ाई 0.50 है तो नमूना आकार कितना बड़ा होना चाहिए।
- iii) एक व्यक्ति ताश के खेल में 500 रुपये की राशि जीतेगा, यदि वह 52 ताश के पत्तों के डेक से एक जैक, एक राजा, एक रानी या एक इक्का निकालता है। यदि खेल खेलने के लिए चार्ज की गई राशि 100 रुपये है, तो क्या खेल खेला जाना चाहिए?
- iv) एक बॉक्स में 5 लाल गेंदें और 10 सफेद गेंदें हैं। एक अन्य बॉक्स में 12 लाल गेंदें और 13 सफेद गेंदें हैं। एक गेंद को दूसरे से यादृच्छया चुना जाता है और पहले डिब्बे में स्थानांतरित कर दिया जाता है। इसके बाद, पहले बॉक्स में से एक गेंद को यादृच्छया चुना जाता है। चुनी गई गेंद के सफेद होने की प्रायिकता क्या है?

Q2. (i) Let  $X$ , the number of flaws on the surface of a randomly selected pump of a certain type, have a Poisson distribution with mean 6.

- Compute  $P(X \leq 8)$
- Compute  $P(5 < X \leq 8)$
- What is the probability that the number of flaws exceeds its mean value by more than one standard deviation?
- If two pumps are independently selected, what is the probability that neither contains a flaw?

ii) Let  $X$  be the number of litres of milk shake that is requested at a café on a hot summer day.

Assume that  $f(x) = \frac{12x(1000-x)^2}{10^{12}}$ ,  $0 < x < 1000$ , zero elsewhere, is the pdf of  $x$ . How many litres of milk shake should the store have ready on each of these days, so that the probability of exhausting its supply on a particular day is 0.05?

iii) The lead content in noodles is assumed to be normally distributed with standard deviation of 0.3. The permissible average lead content in noodles is at most 5.5 percent. To test the true average, 16 independent samples of noodles were analysed and the mean lead content generated was 5.6 percent. Does the sample indicate that the true average percentage of lead content is less than the true mean at 4% level of significance? Also calculate the power of the test if the true mean value is 5.51 percent.

Q2) (i) मान लीजिये कि  $X$ , एक निश्चित प्रकार के बेतरतीब ढंग से चयनित पंप की सतह पर दोषों की संख्या हैं जो कि, माध्य 6 के साथ पॉइसन वितरण है।

क)  $P(X \leq 8)$  की गणना कीजिये।

ख)  $P(5 < X \leq 8)$  की गणना कीजिये।

ग) इसकी क्या प्रायिकता है कि त्रुटियों की संख्या उसके माध्य मान से एक से अधिक मानक विचलन से अधिक है?

घ) यदि दो पंप स्वतंत्र रूप से चुने जाते हैं, तो क्या संभावना है कि किसी भी पंप में कोई खराबी नहीं है?

ii) मान लीजिये कि गर्म गर्मी के दिनों में एक कैफे में जितने लीटर मिल्क शेक का अनुरोध किया जाता है, उसकी संख्या  $X$  है। मान लीजिये कि  $f(x) = \frac{12x(1000-x)^2}{10^{12}}$ ,  $0 < x < 1000$ , zero अन्यथा,  $x$  की पीडीएफ है। स्टोर में प्रत्येक दिन कितने लीटर मिल्क शेक तैयार होना चाहिए, ताकि किसी विशेष दिन इसकी आपूर्ति समाप्त होने की संभावना 0.05 हो?

iii) नूडल्स में लेड की मात्रा सामान्य रूप से 0.3 के मानक विचलन के साथ वितरित मानी जाती है। नूडल्स में अनुमेय औसत लेड सामग्री अधिकतम 5.5 प्रतिशत है। वास्तविक औसत का परीक्षण करने के लिए, नूडल्स के 16 स्वतंत्र नमूनों का विश्लेषण किया गया और औसत लेड सामग्री 5.6 उत्पन्न हुई प्रतिशत थी। क्या नमूना दर्शाता है कि लेड सामग्री का वास्तविक औसत प्रतिशत महत्व के 4% स्तर पर वास्तविक माध्य से कम है? परीक्षण की शक्ति की गणना भी कीजिये यदि सही माध्य मान 5.51 है।

Q3.i) A certain college gives an internal exam in economics and maths to its students. Let  $X$  be the proportion of correct answers a student gets in economics and  $Y$  be the proportion of correct answers he gets in maths, then the joint probability density is given by:

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

- What is the probability that the proportion of correct answer a student will get is less than 0.40 on both the tests?
- What is the probability that the proportion of correct answer a student will get is more than 0.80 on economics test and less than 0.50 on the maths test?
- Are the two variables independent?

ii) a) Consider asymmetric interval where  $\alpha_1 > 0, \alpha_2 > 0, \alpha_1 + \alpha_2 = \alpha$ . Then

$$P \left[ -z_{\alpha_1} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{\alpha_2} \right] = 1 - \alpha$$

Use the above equation to derive a  $100(1-\alpha)\%$  confidence interval for  $\mu$ . Show the steps clearly.

- b) If  $\alpha$  is 0.09 and  $\alpha_1 = \alpha/3, \alpha_2 = 2\alpha/3$ . Find the Confidence interval?
- c) How would your answer to part (b) change if the interval is symmetric.
- d) Which interval would you prefer? Justify your answer.
- e) Also derive a  $100(1-\alpha)\%$  lower confidence bound for  $\mu$ .

Q3) i) एक निश्चित कॉलेज अपने छात्रों को अर्थशास्त्र और गणित में आंतरिक परीक्षा देता है। मान लीजिए कि अर्थशास्त्र में छात्र को मिले सही उत्तरों का अनुपात X है और गणित में उसे मिले सही उत्तरों का अनुपात Y है, तो संयुक्त संभाव्यता घनत्व इस प्रकार दिया जाता है:

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y) & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{अन्यथा} \end{cases}$$

- क) क्या प्रायिकता है कि एक छात्र को दोनों परीक्षाओं में सही उत्तर का अनुपात 0.40 से कम होगा?
- ख) इसकी क्या प्रायिकता है कि एक छात्र के सही उत्तर का अनुपात अर्थशास्त्र की परीक्षा में 0.80 से अधिक और गणित की परीक्षा में 0.50 से कम होगा?
- ग) क्या दो चर स्वतंत्र हैं?

ii) क) असममित अंतराल पर विचार कीजिये जहां  $\alpha_1 > 0, \alpha_2 > 0, \alpha_1 + \alpha_2 = \alpha$ . यह दिया

$$\text{हुआ है कि } P \left[ -z_{\alpha_1} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{\alpha_2} \right] = 1 - \alpha$$

$\mu$  के लिए  $100(1-\alpha)\%$  विश्वास अंतराल प्राप्त करने के लिए उपरोक्त समीकरण का उपयोग कीजिये। कदम स्पष्ट रूप से दिखाएं।

- ख) अगर  $\alpha = 0.09$  है और  $\alpha_1 = \alpha/3, \alpha_2 = 2\alpha/3$  विश्वास अंतराल का ज्ञात कीजिये?
- ग) यदि अंतराल सममित है तो भाग (ख) के लिए आपका उत्तर कैसे बदलेगा?

घ) आप कौन सा अंतराल पसंद करेंगे? आपने जवाब का औचित्य साबित कीजिये।

इ)  $\mu$  के लिए  $100(1-\alpha)\%$  का कम आत्मविश्वास बाध्य प्राप्त कीजिये।

Q4. i) Consider a supply of 1000 pieces of a particular toy into a store. Suppose the store can tolerate about 5% of defective toys. Let  $X$  be the number of defective toys in a sample without replacement of size  $n = 10$ . Suppose the store owner returns the supply back if  $X \geq 2$ . What is the exact distribution that  $X$  will follow? How will you calculate the probability that the store returns the supply of toys which had 5% defective toys? Obtain the probability.

ii) Suppose that travelling time to office is uniformly distributed on  $[0, 4]$  in the morning, while travelling time in the evening from office to work is uniformly distributed on  $[0, 6]$ , independent of the morning travelling time.

- If you travel to office every morning and travel back home in the evening (Monday to Friday) for a week, what is your total expected travel time?
- Assuming independence of travel time, compute the standard deviation of your total travel time.
- What is the expected value and variance of the difference between morning and evening travel times on a given day?
- What are the expected value and variance of the difference between total morning and total evening travel times for a particular week (Monday to Friday)?

iii) Let  $\{x_1, x_2\}$  constitute a random sample from a normal population with mean  $\mu$  and  $\sigma^2 = 4$ . If the null hypothesis  $\mu = \mu_0$  is to be rejected in favour of the alternative hypothesis  $\mu = \mu_1 > \mu_0$  when  $\bar{X} > \mu_0 + 2$ , Find the probability of Type I error.

iv) Let  $X$  denote the number of hours in a day that a randomly selected child watches TV. Suppose the probability density function of  $X$  is

$$f(x) = \frac{\beta}{x^{\beta+1}} \quad x > 1$$

*0 otherwise.*

A random sample of 5 children is taken with the following observation

1.3, 2.4, 1.1, 2.5 and 3.0

Use the method of moments to obtain an estimator of  $\beta$  and then compute the estimate of this data.

Q4) i) एक स्टोर में एक विशेष खिलौने के 1000 टुकड़ों की आपूर्ति पर विचार कीजिये। मान लीजिए कि स्टोर लगभग 5% खराब खिलौनों को सहन कर सकता है। मान लीजिए कि  $n = 10$  के आकार के प्रतिस्थापन के बिना नमूने में दोषपूर्ण खिलौनों की संख्या  $X$  है। मान लीजिए कि स्टोर का मालिक आपूर्ति वापस कर देता है यदि  $X \geq 2$ । सटीक

वितरण क्या है जिसका  $X$  अनुसरण करेगा? आप इस प्रायिकता की गणना कैसे करेंगे कि स्टोर उन खिलाओं की आपूर्ति लौटाता है जिनमें 5% खराब खिलाए थे? संभावना प्राप्त कीजिये।

ii) मान लीजिए कि कार्यालय जाने के लिए यात्रा का समय सुबह  $[0, 4]$  पर समान रूप से वितरित किया जाता है, जबकि शाम को कार्यालय से घर लौटने के लिए यात्रा का समय समान रूप से  $[0, 6]$  पर वितरित किया जाता है, सुबह की यात्रा शाम के समय की यात्रा से स्वतंत्र है।

क) यदि आप रोज सुबह ऑफिस जाते हैं और शाम को घर लौटते हैं तो एक हफ्ते के लिए (सोमवार से शुक्रवार), आपकी कुल यात्रा का अपेक्षित समय क्या है?

ख) यात्रा समय की स्वतंत्रता को मानते हुए, आपके कुल यात्रा समय के मानक विचलन की गणना कीजिये।

ग) किसी दिए गए दिन पर सुबह और शाम की यात्रा के समय के अंतर का अपेक्षित मूल्य और विचरण क्या है?

ड) किसी विशेष सप्ताह (सोमवार से शुक्रवार) के लिए कुल सुबह और कुल शाम की यात्रा के समय के अंतर का अपेक्षित मूल्य और विचरण क्या है?

iii) मान लीजिए  $\{x_1, x_2\}$  एक सामान्य जनसंख्या से माध्य  $\mu$  और  $\sigma^2 = 4$  के साथ एक यादृच्छिक नमूना बनाते हैं। यदि शून्य परिकल्पना  $\mu = \mu_0$  को वैकल्पिक परिकल्पना  $\mu = \mu_1 > \mu_0$  जब  $\bar{X} > \mu_0 + 2$  के पक्ष में खारिज किया जाना है, तब टाइप I त्रुटि की संभावना का पता लगाएं।

iv) मान लीजिये कि  $X$  एक दिन में घंटों की संख्या को निरूपित करता है जब बच्चा टीवी देखता जो एक यादृच्छिक ढंग से चयनित है। मान लीजिए कि  $X$  की संभाव्यता घनत्व फलन है

$$f(x) = \frac{\beta}{x^{\beta+1}} \quad x > 1, \quad 0 \text{ अन्यथा}$$

5 बच्चों का यादृच्छिक नमूना निम्नलिखित अवलोकन के साथ लिया जाता है

1.3, 2.4, 1.1, 2.5 and 3.0

$\beta$  का एक अनुमानक प्राप्त करने के लिए क्षणों (मोमेंट्स) की विधि का उपयोग कीजिये और फिर इस डेटा के अनुमान की गणना कीजिये।

Q5. i) If 23% of all fruit sellers who sell apples prefer buying it on the same day, find approximate probability that among 120 fruit sellers who sell apples, more than 32 will sell it on the same day.

ii) Suppose a particular brand of sanitiser comes in three sizes: 25 ml, 40 ml, and 65 ml. 20% of all purchasers select a 25ml bottle, 50% select 40 ml bottle and the remaining select a 65 ml bottle. Let  $X_1$  and  $X_2$  be the bottle sizes selected by two independently selected customers. Determine the sampling distribution of  $\bar{X}$ . Calculate  $E(\bar{X})$  and compare it to  $\mu$ .

iii) 160 students of BA (H) Economics semester III in Statistics were surveyed and the number of times they visited a particular website on a day was recorded, yielding the following data.

Number of times website visited	Observed Frequency
0	18
1	37
2	43
3	31
4	15
5	9
6	4
7	3

Let  $X$  be the number of times website visited by a randomly chosen student and assume that  $X$  has a Poisson distribution with parameter  $\mu$ .

- Compute an unbiased estimate of  $\mu$  from the above data.
- What is the standard error of the estimator? Also compute the estimated standard error.

iv) For the distribution given below, compute  $E(x)$

$$p(x) = \left(\frac{1}{2}\right)^x, x = 1, 2, 3, \dots, \text{zero elsewhere}$$

Q5) i) यदि सेब बेचने वाले सभी फल विक्रेताओं में से 23% उसी दिन इसे खरीदना पसंद करते हैं, तो लगभग प्रायिकता ज्ञात कीजिए कि सेब बेचने वाले 120 फल विक्रेताओं में से 32 से अधिक उसी दिन इसे बेचेंगे।

ii) मान लीजिए कि एक विशेष ब्रांड का सैनिटाइज़र तीन आकारों में आता है: 25 मिली, 40 मिली और 65 मिली। सभी खरीदारों में से 20% 25 एमएल की बोतल का चयन करते हैं, 50% 40 मिलीलीटर की बोतल का चयन करते हैं और शेष 65 मिलीलीटर की बोतल का चयन करते हैं। मान लीजिये कि  $X_1$  और  $X_2$  दो स्वतंत्र रूप से चुने गए ग्राहकों द्वारा चुने गए बोतल के आकार हैं। एक्स का नमूना वितरण निर्धारित कीजिए।  $E(X)$  की गणना करें और इसकी तुलना  $\mu$  से कीजिए।



iii) सांख्यिकी में बीए (एच) अर्थशास्त्र सेमेस्टर III के 160 छात्रों का सर्वेक्षण किया गया था और एक दिन में वे किसी विशेष वेबसाइट पर कितनी बार गए थे, निम्नलिखित डेटा प्राप्त करते हुए दर्ज किया गया था।

वेबसाइट देखी जाने की संख्या	प्रेक्षित आवृत्ति
0	18
1	37
2	43
3	31
4	15
5	9
6	4
7	3

मान लीजिये कि  $X$  एक यादृच्छिक रूप से चुने गए छात्र द्वारा देखी गई वेबसाइट की संख्या है और मान लीजिये कि  $X$  पैरामीटर  $\mu$  के साथ एक पॉइसन वितरण है।

क) उपरोक्त डेटा से  $\mu$  के निष्पक्ष अनुमान की गणना कीजिए।

ख) अनुमानक की मानक त्रुटि क्या है? अनुमानित मानक त्रुटि की भी गणना कीजिए।

iv) नीचे दिए गए वितरण के लिए,  $E(x)$  की गणना कीजिये:

$$p(x) = \left(\frac{1}{2}\right)^x, x = 1, 2, 3, \dots, \text{zero अन्यथा}$$

Q6 i) Assuming that the runs made by the Indian cricket team in one day matches follow normal distribution. The following is the data on the runs made by the Indian team in ten one day matches.

110, 150, 130, 170, 180, 100, 160, 140, 170, 130.

- Calculate the maximum likelihood estimate of true average runs made.
- Also calculate the maximum likelihood estimate of the true standard deviation of the distribution.
- Estimate the 75<sup>th</sup> percentile of the distribution.

ii) Out of a random sample of 150 students, 108 students said that they opted for GE maths because it is scoring. With 94% confidence what can we say about the maximum error in the estimate of true proportion of students who choose GE maths because it is scoring, irrespective of the sample size?

iii) Two fair six-sided dice are tossed independently. Let  $D$  = the absolute value of the difference between the numbers obtained.

- What is the probability distribution of  $D$ ?
- Determine the cumulative distribution function (cdf) of  $D$ .
- Using the cdf, calculate the probability of  $D$  being at least 3.

Q6) i) यह मानते हुए कि भारतीय क्रिकेट टीम द्वारा एक दिवसीय मैचों में बनाए गए रन सामान्य वितरण का पालन करते हैं। भारतीय टीम द्वारा दस एक दिवसीय मैचों में बनाए गए रनों के आंकड़े निम्नलिखित हैं।

110, 150, 130, 170, 180, 100, 160, 140, 170, 130।

- किए गए वास्तविक औसत रनों के अधिकतम संभावना अनुमान (MLE) की गणना कीजिये।
- वितरण के सही मानक विचलन के अधिकतम संभावना अनुमान (MLE) की गणना भी कीजिये।
- तरण के 75वें प्रतिशतक का अनुमान लगाएं।

ii) 150 छात्रों के यादृच्छिक नमूने में से 108 छात्रों ने कहा कि उन्होंने जीई गणित को चुना क्योंकि यह स्कोरिंग है। नमूना आकार के बावजूद, 94% विश्वास के साथ हम उन छात्रों के सही अनुपात के अनुमान में अधिकतम त्रुटि के बारे में क्या कह सकते हैं जो जीई गणित चुनते हैं क्योंकि यह स्कोरिंग है,?

iii) दो निष्पक्ष छः भुजाओं वाले पासों को स्वतंत्र रूप से उछाला जाता है। मान लीजिए  $D$  = प्राप्त संख्याओं के बीच के अंतर का निरपेक्ष मान।

- $D$  का प्रायिकता बंटन क्या है?
- $D$  के संचयी वितरण समारोह (सीडीएफ) का निर्धारण कीजिये।
- cdf का उपयोग करते हुए,  $D$  के कम से कम 3 होने की प्रायिकता की गणना कीजिये।

**Table A.1 Cumulative Binomial Probabilities**

$$B(x; n, p) = \sum_{y=0}^x b(y; n, p)$$

**a.  $n = 5$**

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.951	.774	.590	.328	.237	.168	.078	.031	.010	.002	.001	.000	.000	.000	.000
	1	.999	.977	.919	.737	.633	.528	.337	.188	.087	.031	.016	.007	.000	.000	.000
	2	1.000	.999	.991	.942	.896	.837	.683	.500	.317	.163	.104	.058	.009	.001	.000
	3	1.000	1.000	1.000	.993	.984	.969	.913	.812	.663	.472	.367	.263	.081	.023	.001
	4	1.000	1.000	1.000	1.000	.999	.998	.990	.969	.922	.832	.763	.672	.410	.226	.049

**b.  $n = 10$**

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.904	.599	.349	.107	.056	.028	.006	.001	.000	.000	.000	.000	.000	.000	.000
	1	.996	.914	.736	.376	.244	.149	.046	.011	.002	.000	.000	.000	.000	.000	.000
	2	1.000	.988	.930	.678	.526	.383	.167	.055	.012	.002	.000	.000	.000	.000	.000
	3	1.000	.999	.987	.879	.776	.650	.382	.172	.055	.011	.004	.001	.000	.000	.000
	4	1.000	1.000	.998	.967	.922	.850	.633	.377	.166	.047	.020	.006	.000	.000	.000
	5	1.000	1.000	1.000	.994	.980	.953	.834	.623	.367	.150	.078	.033	.002	.000	.000
	6	1.000	1.000	1.000	.999	.996	.989	.945	.828	.618	.350	.224	.121	.013	.001	.000
	7	1.000	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.474	.322	.070	.012	.000
	8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.756	.624	.264	.086	.004
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.944	.893	.651	.401	.096

**c.  $n = 15$**

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
	2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
	4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
	5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
	6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
	7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
	8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
	9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
	10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

(continued)

**Table A.1 Cumulative Binomial Probabilities (cont.)**

$$B(x; n, p) = \sum_{y=0}^x b(y; n, p)$$

**d.  $n = 20$**

		<i>p</i>														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
<i>x</i>	0	.818	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.983	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.999	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.997	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000	.000
	5	1.000	1.000	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	1.000	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000	.000
	9	1.000	1.000	1.000	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000	.000
	10	1.000	1.000	1.000	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000	.000
	11	1.000	1.000	1.000	1.000	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000	.000
	12	1.000	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.102	.032	.000	.000	.000
	13	1.000	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.214	.087	.002	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.383	.196	.011	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.585	.370	.043	.003	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.775	.589	.133	.016	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.909	.794	.323	.075	.001
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.976	.931	.608	.264	.017
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.997	.988	.878	.642	.182

(continued)

**Table A.1 Cumulative Binomial Probabilities (cont.)**

$$B(x; n, p) = \sum_{y=0}^x b(y; n, p)$$

e.  $n = 25$

		$p$														
		0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
	0	.778	.277	.072	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.974	.642	.271	.027	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.998	.873	.537	.098	.032	.009	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.966	.764	.234	.096	.033	.002	.000	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.993	.902	.421	.214	.090	.009	.000	.000	.000	.000	.000	.000	.000	.000
	5	1.000	.999	.967	.617	.378	.193	.029	.002	.000	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.991	.780	.561	.341	.074	.007	.000	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	.998	.891	.727	.512	.154	.022	.001	.000	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.953	.851	.677	.274	.054	.004	.000	.000	.000	.000	.000	.000
	9	1.000	1.000	1.000	.983	.929	.811	.425	.115	.013	.000	.000	.000	.000	.000	.000
	10	1.000	1.000	1.000	.994	.970	.902	.586	.212	.034	.002	.000	.000	.000	.000	.000
	11	1.000	1.000	1.000	.998	.980	.956	.732	.345	.078	.006	.001	.000	.000	.000	.000
x	12	1.000	1.000	1.000	1.000	.997	.983	.846	.500	.154	.017	.003	.000	.000	.000	.000
	13	1.000	1.000	1.000	1.000	.999	.994	.922	.655	.268	.044	.020	.002	.000	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.030	.006	.000	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.071	.017	.000	.000	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.149	.047	.000	.000	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.273	.109	.002	.000	.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.439	.220	.009	.000	.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.622	.383	.033	.001	.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.786	.579	.098	.007	.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.904	.766	.236	.034	.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.968	.902	.463	.127	.002
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.993	.973	.729	.358	.026
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.996	.928	.723	.222

**Table A.2 Cumulative Poisson Probabilities**

$$F(x; \mu) = \sum_{y=0}^x \frac{e^{-\mu} \mu^y}{y!}$$

		$\mu$									
		.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
	0	.905	.819	.741	.670	.607	.549	.497	.449	.407	.368
	1	.995	.982	.963	.938	.910	.878	.844	.809	.772	.736
	2	1.000	.999	.996	.992	.986	.977	.966	.953	.937	.920
x	3		1.000	1.000	.999	.998	.997	.994	.991	.987	.981
	4				1.000	1.000	1.000	.999	.999	.998	.996
	5							1.000	1.000	1.000	.999
	6										1.000

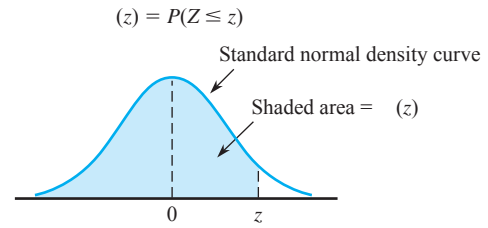
(continued)

**Table A.2 Cumulative Poisson Probabilities (cont.)**

$$F(x; \mu) = \sum_{y=0}^x \frac{e^{-\mu} \mu^y}{y!}$$

	$\mu$										
	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						1.000	.998	.995	.986	.749	.297
18							.999	.998	.993	.819	.381
19							1.000	.999	.997	.875	.470
20								1.000	.998	.917	.559
21									.999	.947	.644
22									1.000	.967	.721
23										.981	.787
24										.989	.843
25										.994	.888
26										.997	.922
27										.998	.948
28										.999	.966
29										1.000	.978
30											.987
31											.992
32											.995
33											.997
34											.999
35											.999
36											1.000

**Table A.3 Standard Normal Curve Areas**



<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas (cont.)

$$\Phi(z) = P(Z \leq z)$$

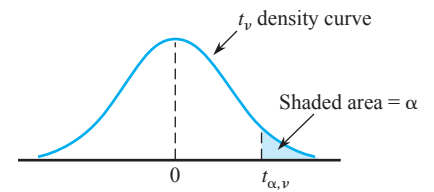
$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998



**Table A.4 The Incomplete Gamma Function**

$$F(x; \alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y} dy$$

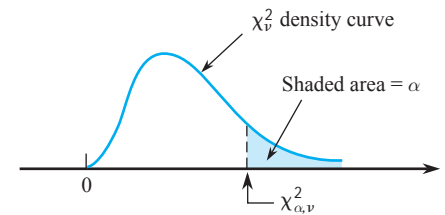
$x \backslash \alpha$	1	2	3	4	5	6	7	8	9	10
1	.632	.264	.080	.019	.004	.001	.000	.000	.000	.000
2	.865	.594	.323	.143	.053	.017	.005	.001	.000	.000
3	.950	.801	.577	.353	.185	.084	.034	.012	.004	.001
4	.982	.908	.762	.567	.371	.215	.111	.051	.021	.008
5	.993	.960	.875	.735	.560	.384	.238	.133	.068	.032
6	.998	.983	.938	.849	.715	.554	.394	.256	.153	.084
7	.999	.993	.970	.918	.827	.699	.550	.401	.271	.170
8	1.000	.997	.986	.958	.900	.809	.687	.547	.407	.283
9		.999	.994	.979	.945	.884	.793	.676	.544	.413
10		1.000	.997	.990	.971	.933	.870	.780	.667	.542
11			.999	.995	.985	.962	.921	.857	.768	.659
12			1.000	.998	.992	.980	.954	.911	.845	.758
13				.999	.996	.989	.974	.946	.900	.834
14				1.000	.998	.994	.986	.968	.938	.891
15					.999	.997	.992	.982	.963	.930

Table A.5 Critical Values for  $t$  Distributions

$\nu$	$\alpha$						
	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
	1.282	1.645	1.960	2.326	2.576	3.090	3.291



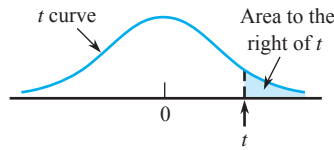
**Table A.7 Critical Values for Chi-Squared Distributions**



$\nu$	$\alpha$									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.971	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.426	65.473
40	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

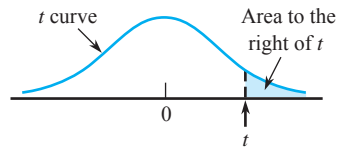
For  $\nu > 40$ ,  $\chi^2_{\alpha, \nu} \approx \nu \left( 1 - \frac{2}{9\nu} + z_{\alpha} \sqrt{\frac{2}{9\nu}} \right)^3$

**Table A.8** *t* Curve Tail Areas



<i>t</i> \ <i>v</i>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
<b>0.0</b>	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
<b>0.1</b>	.468	.465	.463	.463	.462	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461
<b>0.2</b>	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422
<b>0.3</b>	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.384	.384	.384	.384	.384	.384
<b>0.4</b>	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.347	.347	.347	.347	.347
<b>0.5</b>	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.313	.313	.313	.312	.312	.312	.312	.312
<b>0.6</b>	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.278	.278	.278
<b>0.7</b>	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.247	.247	.247	.247	.246
<b>0.8</b>	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.218	.218	.218	.217	.217
<b>0.9</b>	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.191	.191	.191	.190	.190
<b>1.0</b>	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165
<b>1.1</b>	.235	.193	.176	.167	.162	.157	.154	.152	.150	.149	.147	.146	.146	.144	.144	.144	.143	.143
<b>1.2</b>	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.124	.124	.124	.123	.123
<b>1.3</b>	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.105	.105
<b>1.4</b>	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.092	.091	.091	.090	.090	.089
<b>1.5</b>	.187	.136	.115	.104	.097	.092	.089	.086	.084	.082	.081	.080	.079	.077	.077	.077	.076	.075
<b>1.6</b>	.178	.125	.104	.092	.085	.080	.077	.074	.072	.070	.069	.068	.067	.065	.065	.065	.064	.064
<b>1.7</b>	.169	.116	.094	.082	.075	.070	.065	.064	.062	.060	.059	.057	.056	.055	.055	.054	.054	.053
<b>1.8</b>	.161	.107	.085	.073	.066	.061	.057	.055	.053	.051	.050	.049	.048	.046	.046	.045	.045	.044
<b>1.9</b>	.154	.099	.077	.065	.058	.053	.050	.047	.045	.043	.042	.041	.040	.038	.038	.038	.037	.037
<b>2.0</b>	.148	.092	.070	.058	.051	.046	.043	.040	.038	.037	.035	.034	.033	.032	.032	.031	.031	.030
<b>2.1</b>	.141	.085	.063	.052	.045	.040	.037	.034	.033	.031	.030	.029	.028	.027	.027	.026	.025	.025
<b>2.2</b>	.136	.079	.058	.046	.040	.035	.032	.029	.028	.026	.025	.024	.023	.022	.022	.021	.021	.021
<b>2.3</b>	.131	.074	.052	.041	.035	.031	.027	.025	.023	.022	.021	.020	.019	.018	.018	.018	.017	.017
<b>2.4</b>	.126	.069	.048	.037	.031	.027	.024	.022	.020	.019	.018	.017	.016	.015	.015	.014	.014	.014
<b>2.5</b>	.121	.065	.044	.033	.027	.023	.020	.018	.017	.016	.015	.014	.013	.012	.012	.012	.011	.011
<b>2.6</b>	.117	.061	.040	.030	.024	.020	.018	.016	.014	.013	.012	.012	.011	.010	.010	.010	.009	.009
<b>2.7</b>	.113	.057	.037	.027	.021	.018	.015	.014	.012	.011	.010	.010	.009	.008	.008	.008	.008	.007
<b>2.8</b>	.109	.054	.034	.024	.019	.016	.013	.012	.010	.009	.009	.008	.008	.007	.007	.006	.006	.006
<b>2.9</b>	.106	.051	.031	.022	.017	.014	.011	.010	.009	.008	.007	.007	.006	.005	.005	.005	.005	.005
<b>3.0</b>	.102	.048	.029	.020	.015	.012	.010	.009	.007	.007	.006	.006	.005	.004	.004	.004	.004	.004
<b>3.1</b>	.099	.045	.027	.018	.013	.011	.009	.007	.006	.006	.005	.005	.004	.004	.004	.003	.003	.003
<b>3.2</b>	.096	.043	.025	.016	.012	.009	.008	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.002
<b>3.3</b>	.094	.040	.023	.015	.011	.008	.007	.005	.005	.004	.004	.003	.003	.002	.002	.002	.002	.002
<b>3.4</b>	.091	.038	.021	.014	.010	.007	.006	.005	.004	.003	.003	.003	.002	.002	.002	.002	.002	.002
<b>3.5</b>	.089	.036	.020	.012	.009	.006	.005	.004	.003	.003	.002	.002	.002	.002	.002	.001	.001	.001
<b>3.6</b>	.086	.035	.018	.011	.008	.006	.004	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001
<b>3.7</b>	.084	.033	.017	.010	.007	.005	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
<b>3.8</b>	.082	.031	.016	.010	.006	.004	.003	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
<b>3.9</b>	.080	.030	.015	.009	.006	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001
<b>4.0</b>	.078	.029	.014	.008	.005	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.000	.000

(continued)

Table A.8 *t* Curve Tail Areas (cont.)

<i>t</i> \ <i>v</i>	19	20	21	22	23	24	25	26	27	28	29	30	35	40	60	120	$\infty (= z)$
<b>0.0</b>	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
<b>0.1</b>	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.460	.460	.460	.460	.460
<b>0.2</b>	.422	.422	.422	.422	.422	.422	.422	.422	.421	.421	.421	.421	.421	.421	.421	.421	.421
<b>0.3</b>	.384	.384	.384	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.382	.382
<b>0.4</b>	.347	.347	.347	.347	.346	.346	.346	.346	.346	.346	.346	.346	.346	.346	.345	.345	.345
<b>0.5</b>	.311	.311	.311	.311	.311	.311	.311	.311	.311	.310	.310	.310	.310	.310	.309	.309	.309
<b>0.6</b>	.278	.278	.278	.277	.277	.277	.277	.277	.277	.277	.277	.277	.276	.276	.275	.275	.274
<b>0.7</b>	.246	.246	.246	.246	.245	.245	.245	.245	.245	.245	.245	.245	.244	.244	.243	.243	.242
<b>0.8</b>	.217	.217	.216	.216	.216	.216	.216	.215	.215	.215	.215	.215	.215	.214	.213	.213	.212
<b>0.9</b>	.190	.189	.189	.189	.189	.189	.188	.188	.188	.188	.188	.188	.187	.187	.186	.185	.184
<b>1.0</b>	.165	.165	.164	.164	.164	.164	.163	.163	.163	.163	.163	.163	.162	.162	.161	.160	.159
<b>1.1</b>	.143	.142	.142	.142	.141	.141	.141	.141	.141	.140	.140	.140	.139	.139	.138	.137	.136
<b>1.2</b>	.122	.122	.122	.121	.121	.121	.121	.120	.120	.120	.120	.120	.119	.119	.117	.116	.115
<b>1.3</b>	.105	.104	.104	.104	.103	.103	.103	.103	.102	.102	.102	.102	.101	.101	.099	.098	.097
<b>1.4</b>	.089	.089	.088	.088	.087	.087	.087	.087	.086	.086	.086	.086	.085	.085	.083	.082	.081
<b>1.5</b>	.075	.075	.074	.074	.074	.073	.073	.073	.073	.072	.072	.072	.071	.071	.069	.068	.067
<b>1.6</b>	.063	.063	.062	.062	.062	.061	.061	.061	.061	.060	.060	.060	.059	.059	.057	.056	.055
<b>1.7</b>	.053	.052	.052	.052	.051	.051	.051	.051	.050	.050	.050	.050	.049	.048	.047	.046	.045
<b>1.8</b>	.044	.043	.043	.043	.042	.042	.042	.042	.042	.041	.041	.041	.040	.040	.038	.037	.036
<b>1.9</b>	.036	.036	.036	.035	.035	.035	.035	.034	.034	.034	.034	.034	.033	.032	.031	.030	.029
<b>2.0</b>	.030	.030	.029	.029	.029	.028	.028	.028	.028	.028	.027	.027	.027	.026	.025	.024	.023
<b>2.1</b>	.025	.024	.024	.024	.023	.023	.023	.023	.023	.022	.022	.022	.022	.021	.020	.019	.018
<b>2.2</b>	.020	.020	.020	.019	.019	.019	.019	.018	.018	.018	.018	.018	.017	.017	.016	.015	.014
<b>2.3</b>	.016	.016	.016	.016	.015	.015	.015	.015	.015	.015	.014	.014	.014	.013	.012	.012	.011
<b>2.4</b>	.013	.013	.013	.013	.012	.012	.012	.012	.012	.012	.012	.011	.011	.011	.010	.009	.008
<b>2.5</b>	.011	.011	.010	.010	.010	.010	.010	.010	.009	.009	.009	.009	.009	.008	.008	.007	.006
<b>2.6</b>	.009	.009	.008	.008	.008	.008	.008	.008	.007	.007	.007	.007	.007	.007	.006	.005	.005
<b>2.7</b>	.007	.007	.007	.007	.006	.006	.006	.006	.006	.006	.006	.006	.005	.005	.004	.004	.003
<b>2.8</b>	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005	.004	.004	.004	.003	.003	.003
<b>2.9</b>	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.003	.003	.003	.003	.002	.002
<b>3.0</b>	.004	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.001
<b>3.1</b>	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001
<b>3.2</b>	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001
<b>3.3</b>	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000
<b>3.4</b>	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000
<b>3.5</b>	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000
<b>3.6</b>	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000
<b>3.7</b>	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
<b>3.8</b>	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
<b>3.9</b>	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
<b>4.0</b>	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Table A.9 Critical Values for F Distributions

		$\nu_1 = \text{numerator df}$									
		1	2	3	4	5	6	7	8	9	
$\nu_2 = \text{denominator df}$	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
		.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
		.010	4052.20	4999.50	5403.40	5624.60	5763.60	5859.00	5928.40	5981.10	6022.50
		.001	405,284	500,000	540,379	562,500	576,405	585,937	592,873	598,144	602,284
	2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
		.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
		.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
		.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
		.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
		.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
		.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
		.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
		.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
		.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
		.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
		.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
	7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
		.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
		.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
		.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
	8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
		.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
		.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
		.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
	9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
		.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
		.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
		.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
	10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
		.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
		.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
		.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
	11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
		.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
		.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
		.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
		.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
		.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
		.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48

(continued)

Table A.9 Critical Values for F Distributions (cont.)

$\nu_1 = \text{numerator df}$										
10	12	15	20	25	30	40	50	60	120	1000
60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
6055.80	6106.30	6157.30	6208.70	6239.80	6260.60	6286.80	6302.50	6313.00	6339.40	6362.70
605,621	610,668	615,764	620,908	624,017	626,099	628,712	630,285	631,337	633,972	636,301
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36
2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44

(continued)



**Table A.9 Critical Values for F Distributions (cont.)**

		$\nu_1 = \text{numerator df}$									
		1	2	3	4	5	6	7	8	9	
$\nu_2 = \text{denominator df}$	$\alpha$										
	13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
		.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
		.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
		.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
	14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
		.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
		.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
		.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
	15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
		.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
		.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
		.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
	16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
		.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
		.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
		.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
	17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
		.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
		.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
		.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
	18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
		.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
		.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
.001		15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56	
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	
	.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39	
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	
	.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	
21	.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	
	.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	
	.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	
	.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11	
22	.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	
	.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
	.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	
	.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99	
23	.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	
	.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
	.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	
	.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89	
24	.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	
	.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
	.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
	.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80	

(continued)

Table A.9 Critical Values for F Distributions (cont.)

$\nu_1 = \text{numerator df}$										
10	12	15	20	25	30	40	50	60	120	1000
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08
2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87
1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99

(continued)

**Table A.9 Critical Values for F Distributions (cont.)**

		$\nu_1 = \text{numerator df}$									
		1	2	3	4	5	6	7	8	9	
$\nu_2 = \text{denominator df}$	$\alpha$										
	25	.100	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
		.050	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
		.010	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
		.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71
	26	.100	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
		.050	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
		.010	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
		.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
	27	.100	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
		.050	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
		.010	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
		.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57
	28	.100	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
		.050	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
		.010	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
		.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
	29	.100	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
		.050	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
		.010	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
.001		13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	
30	.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	
	.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
	.010	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	
	.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	
40	.100	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	
	.050	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	
	.010	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	
	.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02	
50	.100	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76	
	.050	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	
	.010	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	
	.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82	
60	.100	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	
	.050	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	
	.010	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	
	.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	
100	.100	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	
	.050	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	
	.010	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	
	.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44	
200	.100	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	
	.050	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	
	.010	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	
	.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26	
1000	.100	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64	
	.050	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	
	.010	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	
	.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13	

(continued)

Table A.9 Critical Values for F Distributions (cont.)

$\nu_1 = \text{numerator df}$										
10	12	15	20	25	30	40	50	60	120	1000
1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.61	1.59	1.56	1.52
2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.84	1.82	1.77	1.72
3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.40	2.36	2.27	2.18
4.56	4.31	4.06	3.79	3.63	3.52	3.37	3.28	3.22	3.06	2.91
1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.59	1.58	1.54	1.51
2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.82	1.80	1.75	1.70
3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.36	2.33	2.23	2.14
4.48	4.24	3.99	3.72	3.56	3.44	3.30	3.21	3.15	2.99	2.84
1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.58	1.57	1.53	1.50
2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.81	1.79	1.73	1.68
3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.33	2.29	2.20	2.11
4.41	4.17	3.92	3.66	3.49	3.38	3.23	3.14	3.08	2.92	2.78
1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.57	1.56	1.52	1.48
2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.71	1.66
3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.30	2.26	2.17	2.08
4.35	4.11	3.86	3.60	3.43	3.32	3.18	3.09	3.02	2.86	2.72
1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.56	1.55	1.51	1.47
2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.77	1.75	1.70	1.65
3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.27	2.23	2.14	2.05
4.29	4.05	3.80	3.54	3.38	3.27	3.12	3.03	2.97	2.81	2.66
1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.55	1.54	1.50	1.46
2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.76	1.74	1.68	1.63
2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.25	2.21	2.11	2.02
4.24	4.00	3.75	3.49	3.33	3.22	3.07	2.98	2.92	2.76	2.61
1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.42	1.38
2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.66	1.64	1.58	1.52
2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.06	2.02	1.92	1.82
3.87	3.64	3.40	3.14	2.98	2.87	2.73	2.64	2.57	2.41	2.25
1.73	1.68	1.63	1.57	1.53	1.50	1.46	1.44	1.42	1.38	1.33
2.03	1.95	1.87	1.78	1.73	1.69	1.63	1.60	1.58	1.51	1.45
2.70	2.56	2.42	2.27	2.17	2.10	2.01	1.95	1.91	1.80	1.70
3.67	3.44	3.20	2.95	2.79	2.68	2.53	2.44	2.38	2.21	2.05
1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.41	1.40	1.35	1.30
1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.56	1.53	1.47	1.40
2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.88	1.84	1.73	1.62
3.54	3.32	3.08	2.83	2.67	2.55	2.41	2.32	2.25	2.08	1.92
1.66	1.61	1.56	1.49	1.45	1.42	1.38	1.35	1.34	1.28	1.22
1.93	1.85	1.77	1.68	1.62	1.57	1.52	1.48	1.45	1.38	1.30
2.50	2.37	2.22	2.07	1.97	1.89	1.80	1.74	1.69	1.57	1.45
3.30	3.07	2.84	2.59	2.43	2.32	2.17	2.08	2.01	1.83	1.64
1.63	1.58	1.52	1.46	1.41	1.38	1.34	1.31	1.29	1.23	1.16
1.88	1.80	1.72	1.62	1.56	1.52	1.46	1.41	1.39	1.30	1.21
2.41	2.27	2.13	1.97	1.87	1.79	1.69	1.63	1.58	1.45	1.30
3.12	2.90	2.67	2.42	2.26	2.15	2.00	1.90	1.83	1.64	1.43
1.61	1.55	1.49	1.43	1.38	1.35	1.30	1.27	1.25	1.18	1.08
1.84	1.76	1.68	1.58	1.52	1.47	1.41	1.36	1.33	1.24	1.11
2.34	2.20	2.06	1.90	1.79	1.72	1.61	1.54	1.50	1.35	1.16
2.99	2.77	2.54	2.30	2.14	2.02	1.87	1.77	1.69	1.49	1.22