13/5/16 (Eve)

Roll No.:

Sr. No. of Question Paper: 6485 F-6

Unique Paper Code: 2362801

Name of the Course: B.Sc. (H) Statistics (Allied Course)

Name/Title of the Paper: Operational Research: Optimization Technique

Semester: IV

Duration: 3 hours.

Max. Marks: 75

Question No. 1 is compulsory

Attempt five questions in all.

Use of non-scientific calculator is permitted.

Q1 a) Obtain the necessary and sufficient conditions for the optimum solution of the following NLPP:

Max $z = 3.6\, x_1 - 0.4\, x_1^2 + 1.6\, x_2 - 0.2\, x_2^2$

subject to the constraints

$2x_1 + x_2 = 10, \quad x_1, x_2 \geq 0$

b) Use dynamic programming to

Maximize $z = y_1 . y_2 . y_3$

subject to the constraints

$y_1 + y_2 + y_3 = 5$

$y_1, y_2, y_3 \geq 0$

c) The estimated sales of proposed types of perfumes are as under:

	Estimated levels of sales(units)		
Types of perfumes	High	Medium	Low
A	25	15	10
B	40	20	5
C	60	25	3

For each of the following decision criterion, state the optimal action and specify the value leading to its selection:

(i) Maximin (ii) Maximax.

d) Formulate the following LPP as a goal programming problem:

Min $(2\, x_1 - x_2,\ 4x_1 - 5x_2,\ -x_1)$

subject to the constraints

$4\, x_1 + 5\, x_2 \leq 20$

$3\, x_1 + 2\, x_2 \leq 12$

$x_1, x_2 \geq 0.$

The decision maker decided to give first priority to the second objective criterion, second priority to the third objective and third priority to the first objective criterion. He also wishes to keep the first, second and third priorities objective values 8, -2, 1 respectively.

e) Describe a dynamic programming problem and discuss its applications. (5x5=25)

Q2) Define Bellman's Principle of Optimality.

A 5-ton vessel can be loaded with one or more of the three items. The following table gives the unit weight w_i (in tons) and the unit revenue (in thousands of dollars) r_i for item i. How the vessel should be loaded to maximize the total revenue?

Item i	Weight(w_i)	Revenue (r_i)
1	2	31
2	3	47
3	1	14

$(12\frac{1}{2})$

Q3) Use Wolfe's method to solve the following quadratic programming problem:

$$\text{Max } z = 4x_1+6x_2-x_1^2-2\,x_1x_2-2\,x_2^2$$

subject to the constraints :

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

$(12\frac{1}{2})$

Q4 a) A farmer wants to decide which of the three crops he should plant on his 100- acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high, medium and low. His estimated profit for each is shown in the table below:

Rainfall	Estimated conditional profit (in Rs.)		
	Crop A	Crop B	Crop C
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only one crop, decide which should be his best crop using,

i) Hurwicz criterion (farmer's degree of optimism being 0.6)
ii) Laplace criterion
iii) Minimax regret criterion.

(6)

b) A farmer can plant either corn or beans. The probabilities that the next harvest prices of these commodities will go up, stay the same or go down are 0.25, 0.30 and 0.45 respectively. If the prices go up, the corn crop will net rupees 30,000 and the beans will net

rupees 10,000. If the prices remain unchanged, the farmer will barely break even. But, if the prices go down, the corn and beans crops will sustain losses of rupees 35,000 and rupees 5,000 respectively.

i) Represent the farmer's problem as a decision tree.
ii) Which crop should the farmer plant? $(6\frac{1}{2})$

Q5 a) Use Dynamic Programming to find the minimum value of

$$x_1^2 + 2x_2^2 + 4x_3$$

subject to the constraints:

$$x_1 + 2x_2 + x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0.$$ (6)

b) An office equipment company manufactures two kinds of products: chairs and lamps. Production of either a chair or a lamp requires 1 hour of production capacity in the plant. The plant has a maximum production capacity of 50 hours a week. Because of limited sales capacity, the maximum numbers of chairs and lamps that can be sold are 6 and 8 per week respectively. The gross margin from the sale of chair is rupees 30 and rupees 60 for a lamp. The plant manager desires to determine the number of units of each product that should be produced per week in consideration of the following set of goals:

i) Available production capacity should be fully utilized and not exceeded.
ii) Sales of two products should be as much as possible since the gross margin from sale of a lamp is twice the amount from a chair. The company has twice as much desire to achieve sales for lamps as for chairs.
iii) Minimize the overtime operation of the plant as much as possible.

Formulate this problem as a goal programming problem so that the manager achieves his goals as closely as possible. $(6\frac{1}{2})$

Q6) a) Solve the following non- linear programming problem:

Maximize $z = 2x_1^2 + 12x_1x_2 - 7x_2^2$

subject to the constraints:

$$2x_1 + 5x_2 \leq 98$$

and $x_1, x_2 \geq 0$

b) Solve the following goal programming problem using graphical method: (6)

Minimize $Z = (P_1 d_1^+ + P_1 d_2^+) + (30 P_2 d_3^- + 40 P_2 d_4^-) + P_3 d_5^-$

subject to :

$$2x_1 + 4x_2 + d_1^- - d_1^+ = 80$$

$$3x_1 + 3x_2 + d_2^- - d_2^+ = 80$$

$$x_1 + d_3^- - d_3^+ = 10$$

$$x_2 + d_4^- - d_4^+ = 10$$

$$30x_1 + 40x_2 + d_5^- - d_5^+ = 1200$$

$x_1, x_2, d_i^-, d_i^+ \geq 0$; i = 1, 2, 3, 4, 5. $(6\frac{1}{2})$

Q7) Write short notes on the following:

i) Goal Programming

ii) Convex function and concave function

iii) Quadratic Programming $(4\frac{1}{2}, 4, 4)$