

1/12 / 16 (Eve)

This question paper contains 4 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--

S. No. of Question Paper : 1415

Unique Paper Code : 2341504

F-7

Name of the Paper : Mathematical Physics—II

Name of the Course : B.Tech. (Computer Science) Allied Course

Semester : V

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do Five questions in all.

Question No. 1 is compulsory.

1. Do any five questions :

5×3=15

(a) Classify (order, degree, linear/non-linear) the following differential equation :

$$5\left(\frac{d^3y}{dx^3}\right)^4 + 3x^2\left(\frac{d^2y}{dx^2}\right)^5 + 4x\left(\frac{dy}{dx}\right)^7 + y = x^3.$$

(b) Check whether the following functions are linearly dependent or independent :

$$e^x \sin x, e^x \cos x.$$

(c) Prove the following property of Poisson Bracket :

$$[a, bc] = b[a, c] + [a, b]c.$$

P.T.O.

(d) Find the extreme points of the function :

$$f(x, y) = 4xy - x^4 - y^4.$$

(e) Solve :

$$(x^2 + y^2)dx - 2xydy = 0.$$

(f) Define generalised momenta for n -particle system, and find its time derivative.

(g) Form the differential equation whose only solutions are :

$$a_1 e^x, a_2 e^{2x}, a_3 e^{3x}.$$

(h) Find the extremal of the integral :

$$\int_0^{\pi} (y'^2 - y^2) dx, \text{ here } y' = \frac{dy}{dx}.$$

2. Solve the following differential equations :

(a) $x^4 \frac{dy}{dx} + x^3 y = -\sec(xy)$ 6

(b) $\frac{dy}{dx} = \frac{y + x - 2}{y - x - 4}$ 9

3. Solve the following differential equations :

(a) $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^{2x} (x^3 + \cos 3x)$ 6

(b) $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\ln x)$ 9

4. (a) Solve the following differential equation : 6

$$(x^3 + y^3)dx - 2xy^2dy = 0.$$

- (b) Using the method of variation of parameters, solve : 9

$$(D^2 + 9)y = x \cos 3x; D \equiv \frac{d}{dx}.$$

5. (a) Using the method of undetermined coefficients, solve : 6

$$(D^2 + 1)y = 2e^x + \cos x; D \equiv \frac{d}{dx}.$$

- (b) Solve the coupled differential equations : 9

$$\frac{dx}{dt} + \frac{dy}{dt} - x = 2t + 1$$

$$2\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + x = t.$$

6. (a) Prove that the equation of the shortest path between two points on the surface of right circular cylinder of radius a is given by :

$$z = c_1\phi + c_2,$$

where c_1 and c_2 are constants. 6

- (b) Using Lagrange's method of undetermined multiplier, find the area of the largest triangle inscribed in the circle of radius a . 9

7. (a) Find the Hamiltonian corresponding to the Lagrangian : 6

$$L = ax^2 + by^2 - kxy.$$

- (b) Starting from the expression :

$$H(q, p) = p\dot{q} - L(q, \dot{q})$$

Derive the Hamilton's equations of motion.

9

8. (a) Show that the time derivative of $u(q, p)$ is given by :

$$\frac{du}{dt} = [u, H]$$

here, H denotes Hamiltonian.

6

- (b) Write the Lagrangian of the system of two masses m and $2m$, shown below in Fig. 1. In this figure, x_1 and x_2 are the displacements of two masses from their equilibrium positions. Hence obtain the equations of motion of these two masses.

9

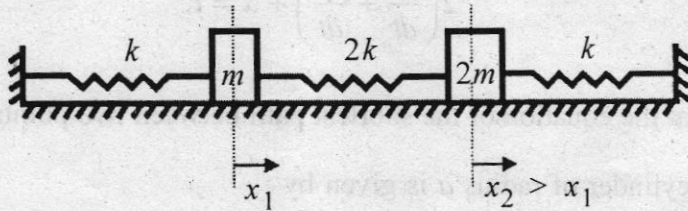


Fig. 1