03/06/26

This question paper contains 4 printed pages]

**Your Roll No.** : ......................

**Sl. No. of Q. Paper** : **1932** **I**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus-DSC

Name of the Course : **B.Sc. (H) Mathematics**

Semester : IV

***Time : 3 Hours*** ***Maximum Marks : 90***

**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) **All** questions are compulsory and are of **15** marks each.

(c) Attempt any **two** parts from each Questions. Each part is of **7.5** marks.

**1.** (a) Consider the function:

$$f(x,y)=\begin{cases}\dfrac{2x^2-x^2y^2+2y^2}{x^2+y^2} & \text{for } (x,y)\neq(0,0)\\ A & \text{for } (x,y)=(0,0)\end{cases}$$

Find the value of A, which will make f continuous at the (0,0).

(b) The temperature at a point (x, y) on a given metal plate in the xy plane is determined according to the formula $T(x,y) = x^3+2xy^2+y$ degrees. Compute the rate at which the temperature changes with distance if we start at (2,1) and move

(i) parallel to the vector j

(ii) parallel to the vector i



P.T.O.

(c) A juice can is 12 cm tall and has a radius of 3 cm. A manufacturer is planning to reduce the height of the can by 0.2 cm and the radius by 0.3 cm. Use a total differential to estimate the percentage decrease in volume that occurs when the new cans are introduced. (Round to the nearest percent.)

**2.** (a) Let $w = f(t)$ be a differentiable function of t, where $t = (x^2+y^2+z^2)^{1/2}$. show that

$$\left(\frac{dw}{dt}\right)^2 = \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2$$

(b) Find the directional derivative of $f(x,y) = e^{x^2} y^2$ at $P_0(1, -1)$ in the direction toward $Q\ (2,3)$.

(c) Find the absolute extrema of the function $f(x,y) = xy-2x-5y$ on the closed bounded set S, where S is the triangular region with vertices (0,0) (7,0) and (7,7).

**3.** (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy-plane and above by the graph:

$$z = \frac{xy}{\sqrt{x^2+y^2+1}}; \qquad R: 0 \le x \le 1, 0 \le y \le 1$$

(ii) Sketch the region of integration and compute the double integral:

$$\int_0^{\frac{\pi}{3}} \int_0^{y^2} \frac{1}{y} \sin\frac{x}{y}\, dx dy$$

(b) (i) Compute double integral by converting to polar coordinates:

$$\int_0^3 \int_x^{\sqrt{9-x^2}} \cos\left(x^2+y^2\right) dydx$$

(ii) Use a double integral to find the area bounded by $r = \cos 2\theta$ .

(c) (i) Evaluate Triple integral:

$$\iiint_D x dV$$

where, D is bounded by the paraboloid $z = x^2+y^2$ and the planc $z = 1$.

(ii) Compute the iterated triple integral.

$$\int_{-1}^{2} \int_0^{\pi} \int_1^4 yz \cos xy dzdxdy$$

4. (a) Find the volume of the solid bounded by cylinders $y = z^2$ and $y = 2 - z^2$ and the planes $x = 1$ and $x = -2$

(b) Evaluate triple integral

$$\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$$

where, D is the solid sphere $x^2+y^2+z^2 \leq 3$

(c) Use suitable change of variable find the area of the region R bounded by the hyperbolas $xy = 1$ and $xy = 4$ and the lines $y = x$ and $y = 4x$.

5. (a) Evaluate $\int_C F\, dR$, where $F = (y^2-z^2)i+(2yz)j-x^2k$ and C is the curve defined parametrically by $x = t^2$, $y = 2t$ and $z = t$, for $0 \leq t \leq 1$.

(b) Show that the vector field $F(x,y) = (x+2y)i+(2x+y)j$ is conservative and find a scalar potential f for F. Evaluate the line integral $\int_C F\, dR$, where C is any smooth path connecting A(0,0) to B(1,1).

(c) Use Green's theorem to find the work done by the force field

$F(x,y) = (3y-4x)i+(4x-y)j$

When an object moves once counterclockwise around the ellipse $4x^2+y^2=4$.

6. (a) Evaluate $\iint_S (x^2+y^2)\, dS$, where S is the surface of the hemisphere $z=\sqrt{1-x^2-y^2}$.

(b) Let S be the portion the plane $x+y+z = 1$ that lies in the first octant, and let C be the boundary of S, traversed counterclockwise. Verify Stokes' theorem for the surface S and the vector field F=

$$F=-\frac{3}{2}y^2 i-2xyj+yzk.$$

(c) Use the divergence theorem to evaluate.

$$\iint_S F.NdS$$

Where, $F = xyzi + xyzj + xyzk$, S is the surface of the box $0\le x\le 1, 0\le y\le 2, 0\le z\le 3$, with outward unit normal vector N.

***