28/5/26

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **1793**

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : **B.Sc. (H) Mathematics**

Semester : II / DSC

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculators is not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that

$$\| x + y \| \geq | \| x \| - \| y \| |$$

Also verify it for the vectors $x = [2, -1, 3, 2]$ and $y = [4, 3, 2, 1]$ in $\mathbb{R}^4$. (5+2.5)

(b) Solve the following system of linear equations using Gaussian Elimination method :

$$3x_1 + x_2 + 7x_3 + 2x_4 = 13$$

$$2x_1 - 4x_2 + 14x_3 - x_4 = -10$$

$$5x_1 + 11x_2 - 7x_3 + 8x_4 = 59$$

$$2x_1 + 5x_2 - 4x_3 - 3x_4 = 39$$

Also give the complete solution set if the system is consistent. (6+1.5)

(c) Using the Gauss-Jordan method, find the minimal integer values for the variables that will balance the following chemical equation :

$a(H_3PO_4) + b(Ca(OH)_2) \rightarrow c(Ca_3(PO_4)_2) + d(H_2O)$ (7.5)

2. (a) Find the reduced row echelon form B of the following matrix A, keeping track of the row operations used :

$$A = \begin{bmatrix} 4 & 0 & -20 \\ -2 & 0 & 11 \\ 3 & 1 & -15 \end{bmatrix}$$

(7.5)

(b) Determine whether the vector x = [5,17, –20] is in the row space of the matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$$

If so, then express [5,17, –20] as a linear combination of the rows of A. (5+2.5)

(c) (i) Define Eigenvalue and Eigenspace for a square matrix A.

(ii) Find all the eigenvalues and eigenspaces for A.

$$A = \begin{bmatrix} -4 & 8 & -12 \\ 6 & -6 & 12 \\ 6 & -8 & 14 \end{bmatrix}$$

(2+5.5)

3. (a) Define a subspace of a vector space over $\mathbb{R}$. Which of the following are subspaces of $M_{2\times2}(\mathbb{R})$? Give reasons to support your answer.

(i) $W_1 = \{A \in M_{2\times2}: \text{Trace}(A) = 0\}$

(ii) $W_2 = \{A \in M_{2\times2}: A^T = 4\}$

(iii) $W_3 = \{A \in M_{2\times2}: |A| = 0\}$ (1.5+2+2+2)

(b) Define sum of two subspaces W_1 and W_2 of a vector space $V(\mathbb{R})$. Show that $W_1 + W_2$ is the smallest subspace of V containing both W_1 and W_2. (2+5.5)

(c) Define Linear Span of a set S in a vector space V. Is the linear span of S a subspace of V? Determine if the vector $(2, -1, 1)$ is in the linear span of $S = \{(1,0,2), (-1,1,1)\}$ (2+2+3.5)

4. (a) Define a linearly independent subset of a vector space. For $k = 0,1,2,\ldots, n$, $p_k(x) = x^k + x^{k+1} + \cdots + x^n$. Show that $\{p_0(x), p_1(x), \ldots, p_1(x)\}$ is linearly independent subset of the space of all polynomials. (2+5.5)

(b) Let $W = \{(a_1, a_2, a_3, a_4, a_5): a_2 = a_4, a_1 + a_3 + a_5 = 0\}$ be a subset of $\mathbb{R}^5$. Is W a subspace of $\mathbb{R}^5$? Find a basis for W. What is $\dim_{\mathbb{R}} W$? (2+4+1.5)

(c) Let $S = \{(1,3,-4,2), (2,2,-4,0), (1,-3,2,-4), (-1,0,1,0)\}$ be a subset of $\mathbb{R}^4$. Show that S is linearly dependent. Write a vector in S as a linear combination of other vectors. (5.5+2)

5. (a) Define a linear transformation. Is there any linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(1,0,3) = (1,1)$ and $T(-2,0,-6) = (2,1)$? (2+5.5)

(b) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(a, b) = (a - b, a, 2a + b)$

Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$. Compute $[T]_\beta^\gamma$. (7.5)

(c) (i) Let V and W be vector spaces and let $T : V \to W$ be a linear transformation. Then show that T is one-to-one if and only if $N(T) = \{0\}$.

(ii) Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation defined by

$$T(f(x)) = 2f'(x) + \int_0^x 3f(t)dt.$$

Show that T is one-one. (3+4.5)

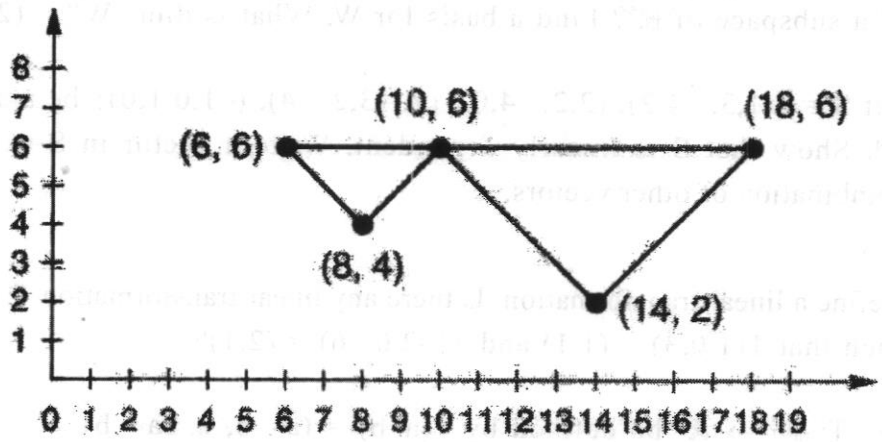
6. (a) Let V and W be vector spaces and let $T: V \to W$ be a linear. Define inverse of T. If T is invertible linear transformation then show that $T^{-1}: W \to V$ is linear. (2+5.5)

(b) Define isomorphic vector spaces. Which of the following spaces are isomorphic? Justify your answer.

(i) $M_{2\times2}(\mathbb{R})$ and $P_3(\mathbb{R})$

(ii) $V = \{A \in M_{2\times2}(\mathbb{R}): \text{tr}(A) = 0\}$ and $\mathbb{R}^4$ (2.5+2.5+2+2.5)

(c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



(i) Rotation about the origin through $\theta = 90°$

(ii) Reflection about the line $y = \frac{1}{2}x$.

(iii) Translation along the vector $[-3, 5]$ (2.5+2.5+2.5)