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This question paper contains 4 printed pages]

Your Roll No.

:

Sl. No. of Q. Paper

: **2058**

Unique Paper Code

: **2352011202**

Name of the Paper

: **DSC-5: Calculus**

Name of the Course

: **B.Sc. (H) Mathematics
(NEP-UGCFC 2022)**

Semester

: **II**

Time : 3 Hours

Maximum Marks : 90

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- Attempt **all** questions by selecting **three** parts from each question.
- The parts of each question are to be attempted together.
- All** questions carry equal marks.
- Use of Calculator is **not** allowed.

- (a) Let $f: A \rightarrow R$ and let c be a cluster point of A . Show that $\lim_{x \rightarrow c} f(x) = L$ if and only if for every sequence $\langle x_n \rangle$ in A that converges to c such that $x_n \neq c$ for all $n \in N$, the sequence $\langle f(x_n) \rangle$ converges to L .

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- (b) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist in $\mathbb{R}$.
- (c) Using $\epsilon - \delta$ definition, show that $\lim_{x \rightarrow 1} \frac{x}{1+x} = \frac{1}{2}$.
- (d) Find $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1+3x}}{x+2x^2}$ where $x > 0$.
2. (a) Suppose that the functions f and g have limits in $\mathbb{R}$ as $x \rightarrow \infty$ and that $f(x) \leq g(x)$ for all $x \in (0, \infty)$. Prove that $\lim_{x \rightarrow \infty} f \leq \lim_{x \rightarrow \infty} g$.
- (b) Prove by verifying the $\epsilon - \delta$ property that the function $f(x) = x^2$ is continuous at $x_0 = 2$.
- (c) State the Intermediate Value Theorem and show that $x = \cos x$ for some x in $\left(0, \frac{\pi}{2}\right)$.
- (d) Show that the function $f(x) = \frac{1}{x^2}$ is not uniformly continuous on $(0, 1)$.
3. (a) Prove that every uniformly continuous function on a set A is continuous on A . Is the converse true? Justify.

5. (a) If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, prove that $I_n = n! I_{n-1} + (n-1)!$ and hence prove that

$$I_n = n! \left[\frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{2} + 1 + \log x \right].$$

- (b) If $y = e^{a \sin^{-1} x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$$

- (c) State the Taylor's theorem for a function defined on some open interval I of real numbers. Also, find the Taylor's series expansion of e^{2x} , for $x \in \mathbb{R}$ about zero.

- (d) Sketch a graph of the equation $y = \frac{2x^2 - 8}{x^2 - 16}$.

6. (a) Find $\lim_{x \rightarrow 0} (1 + \sin x)^x$.

- (b) Find the asymptotes of the rational

$$\text{function, } y = \frac{x^3 - x^2 - 8}{x-1}$$

- (c) Examine the nature of origin on the curve.

$$(2x+y)^2 - 6xy(2x+y) - 7x^3 = 0$$

- (d) Trace the curve $r = 1 - \cos \theta$.

- (b) If f and g are functions that are differentiable at the point a , then prove that $(fg)'(a) = f(a)g'(a) + f'(a)g(a)$.
- (c) Let $f(x) = |x| + |x+1|$ for $x \in \mathbb{R}$. Determine the set of points at which the function f is not differentiable. Justify your answer.
- (d) Prove $|\cos x - \cos y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.
4. (a) Suppose that $f: [0, 2] \rightarrow \mathbb{R}$ is continuous on $[0, 2]$, differentiable on $(0, 2)$ and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.
- Show that $f'(c_1) = 1$ for some $x \in (0, 1)$.
 - Show that $f'(c_2) = 0$ for some $x \in (1, 2)$.
 - Show that $f'(c) = \frac{1}{10}$ for some $x \in (0, 2)$.
- (b) State and prove Mean Value Theorem.
- (c) Show that $e^x \geq 1 + x$ for all $x \in \mathbb{R}$.
- (d) Prove that if f and g are differentiable on $\mathbb{R}$, if $f(0) = g(0)$ and if $f'(x) \leq g'(x)$ for all $x \in \mathbb{R}$, then $f(x) \leq g(x)$ for all $x \geq 0$.