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This question paper contains 4 printed pages]

Roll No.

S. No. of Question Paper : **2039**

Unique Paper Code : **2352012403**

Name of the Paper : **DSC-Numerical Analysis**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of scientific calculator is allowed.

1. (*a*) Define the order of convergence of a sequence p_n which converges to a number p. Show that the sequence generated by the formula

$$p_{n+1} = \frac{p_n^3 + 3p_n a}{3p_n^2 + a}$$

converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.

P.T.O.

(*b*) Verify that the equation $x^3 + x^2 - 3x - 3 = 0$ has a root on the interval (1, 2). Perform the bisection method to determine p_4, the fourth approximation to the location of the root.

(*c*) Perform four iterations of the method of false position to approximate the root of the equation $x^4 - 18x^2 + 45 = 0$ in the interval (1, 2). 15

2. (*a*) Let g be a continuous function on the interval $[a, b]$ such that $g([a, b]) \subset [a, b]$. Suppose g is differentiable function on (a, b) and satisfies

$$|g'(x)| \leq k < 1, \forall\, x \in (a, b)$$

A sequence p_n is generated by the fixed point iteration $p_n = g(p_{n-1})$. $n \geq 1$. Show that the sequence $\{p_n\}$ converges to a fixed point p of g for any initial approximation $p_0 \in [a, b]$.

(*b*) Use Newton's method to approximate the root of the equation $x^5 + 2x - 1 = 0$ in the interval (0, 1). Use error tolerance 10^{-4}, and initial approximation $p_0 = 0.5$.

(*c*) Perform the Secant method to determine p_4, the fourth approximation to the location of the root of the equation $\tan(\pi x) - \pi - 6 = 0$ with starting approximations $p_0 = 0$, $p_1 = 0.48$. 15

3. (*a*) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [14 \quad 36 \quad 7]^T$.

(*b*) Perform three iterations of Jacobi method to solve the system of equations, for the given coefficient matrix and right hand side vector, starting with the initial vector $x^{(0)} = 0$,

$$\begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}$$

(*c*) State and prove existence and uniqueness theorem for Interpolating Polynomial. 15

4. (*a*) Find the Lagrange form of interpolating polynomial for the given data set and hence interpolate at $x = 1.5$.

x	-1	0	1	2
$f(x)$	5	1	1	11

(*b*) If $f(x) = \frac{1}{x}$ then evaluate the n^{th} divided difference $f[x_0, x_1, x_2, ..., x_n]$.

(*c*) Obtain the piecewise Linear Interpolating Polynomial for the function $f(x)$ defined by the given data set and hence estimate the value of $f(1.5)$

x	0	1	2	3	4
$f(x)$	1	2	5	10	16

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5. (a) Prove the forward difference approximation formula $f'(x_0) \approx \dfrac{-3f(x_0)+4f(x_0+h)+f(x_0+2h)}{2h}$ and determine the approximate value of the derivative of $f(x) = 2x^2 - 1$ at $x_0 = 1$, with the step length h = 0.1, 0.01 and 0.001, and absolute error.

(b) Verify that the second-order central difference approximation for the second derivative provides the exact value of the second derivative, regardless the value of h, for the functions $f(x) = 1, f(x) = x, f(x) = x^2$, and $f(x) = x^3$, but not for the function $f(x) = x^4$.

(c) Derive the expression of the Simpson's rule to approximate definite integral and hence approximate the value of the integral $\int_0^1 e^{-x}\,dx$. 15

6. (a) Define the degree of precision of the quadrature rule. Further, calculate the values of the coefficients A_0, A_1, and A_2 so that the quadrature formula $I(f) = \int_{-1}^{1} f(x)dx = A_0 f\left(\dfrac{-1}{2}\right) + A_1 f(0) + A_2 f\left(\dfrac{1}{2}\right)$ has degree of precision at least 2.

(b) Use the Modified Euler Method to determine the approximate solution of the initial value problem $\dfrac{dx}{dt} = \dfrac{t}{x}$, $x(1) = 1, 1 \le t \le 2$, taking the step size as h = 0.5.

(c) Use the Runge-Kutta Method fourth order to determine the approximate solution of the initial value problem $\dfrac{dx}{dt} = 1 + \dfrac{x}{t}$, $x(1) = 1, 1 \le t \le 2$, taking the step size as h = 0.5. 15