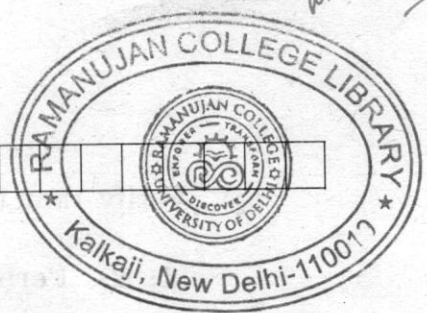


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S. No. of Question Paper : **2039**

Unique Paper Code : **2352012403**

Name of the Paper : **DSC-Numerical Analysis**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours**

Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of scientific calculator is allowed.

1. (a) Define the order of convergence of a sequence p_n which converges to a number p . Show that the sequence generated by the formula

$$p_{n+1} = \frac{p_n^3 + 3p_n a}{3p_n^2 + a}$$

converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.

P.T.O.

(b) Verify that the equation $x^3 + x^2 - 3x - 3 = 0$ has a root on the interval $(1, 2)$. Perform the bisection method to determine p_4 , the fourth approximation to the location of the root.

(c) Perform four iterations of the method of false position to approximate the root of the equation $x^4 - 18x^2 + 45 = 0$ in the interval $(1, 2)$. 15

2. (a) Let g be a continuous function on the interval $[a, b]$ such that $g([a, b]) \subset [a, b]$. Suppose g is differentiable function on (a, b) and satisfies

$$|g'(x)| \leq k < 1, \forall x \in (a, b)$$

A sequence p_n is generated by the fixed point iteration $p_n = g(p_{n-1})$, $n \geq 1$.

Show that the sequence $\{p_n\}$ converges to a fixed point p of g for any initial approximation $p_0 \in [a, b]$.

(b) Use Newton's method to approximate the root of the equation $x^5 + 2x - 1 = 0$ in the interval $(0, 1)$. Use error tolerance 10^{-4} , and initial approximation $p_0 = 0.5$.

(c) Perform the Secant method to determine p_4 , the fourth approximation to the location of the root of the equation $\tan(\pi x) - \pi - 6 = 0$ with starting approximations $p_0 = 0$, $p_1 = 0.48$. 15

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [14 \ 36 \ 7]^T$.

- (b) Perform three iterations of Jacobi method to solve the system of equations, for the given coefficient matrix and right hand side vector, starting with the initial vector $x^{(0)} = 0$,

$$\begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}$$

- (c) State and prove existence and uniqueness theorem for Interpolating Polynomial. 15

4. (a) Find the Lagrange form of interpolating polynomial for the given data set and hence interpolate at $x = 1.5$.

x	-1	0	1	2
$f(x)$	5	1	1	11

- (b) If $f(x) = \frac{1}{x}$ then evaluate the n^{th} divided difference $f[x_0, x_1, x_2, \dots, x_n]$.
- (c) Obtain the piecewise Linear Interpolating Polynomial for the function $f(x)$ defined by the given data set and hence estimate the value of $f(1.5)$

x	0	1	2	3	4
$f(x)$	1	2	5	10	16

15

5. (a) Prove the forward difference approximation formula $f'(x_0) \approx \frac{-3f(x_0) + 4f(x_0 + h) + f(x_0 + 2h)}{2h}$ and determine the approximate value of the derivative of $f(x) = 2x^2 - 1$ at $x_0 = 1$, with the step length $h = 0.1, 0.01$ and 0.001 , and absolute error.
- (b) Verify that the second-order central difference approximation for the second derivative provides the exact value of the second derivative, regardless the value of h , for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$, but not for the function $f(x) = x^4$.
- (c) Derive the expression of the Simpson's rule to approximate definite integral and hence approximate the value of the integral $\int_0^1 e^{-x} dx$. 15
6. (a) Define the degree of precision of the quadrature rule. Further, calculate the values of the coefficients A_0 , A_1 , and A_2 so that the quadrature formula $I(f) = \int_{-1}^1 f(x) dx = A_0 f\left(\frac{-1}{2}\right) + A_1 f(0) + A_2 f\left(\frac{1}{2}\right)$ has degree of precision at least 2.
- (b) Use the Modified Euler Method to determine the approximate solution of the initial value problem $\frac{dx}{dt} = \frac{t}{x}$, $x(1) = 1$, $1 \leq t \leq 2$, taking the step size as $h = 0.5$.
- (c) Use the Runge-Kutta Method fourth order to determine the approximate solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $x(1) = 1$, $1 \leq t \leq 2$, taking the step size as $h = 0.5$. 15