

This question paper contains 4 printed pages]

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S. No. of Question Paper : 5788

Unique Paper Code : 2372011201

Name of the Paper : Theory of Probability Distributions (NEP)

Name of the Course : B.Sc. (Hons.) Statistics (DSC)

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all, selecting three questions from each section.

All questions/parts are of equal marks.

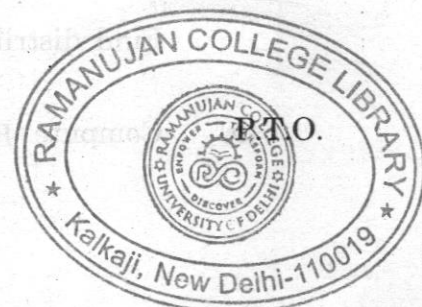
The symbols used have their usual meaning.

Use of non-programmable scientific calculator is allowed.

### Section-A

1. (a) Suppose a two-dimensional continuous random variable (X, Y) has joint p.d.f. given by

$$f(x, y) = \begin{cases} 6x^2y; & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$



Find :

$$P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right),$$

$$P(X + Y < 1) \text{ and } P(X < 1 | Y < 2).$$

(b) Define the cumulant generating function and show that :

$$k_1 = \mu'_1, k_2 = \mu_2, k_3 = \mu_3 \text{ and } k_4 = \mu_4 + 3\mu_2^2.$$

2. (a) Let

$$f(x, y) = \begin{cases} 8xy; & 0 < x < y < 1 \\ 0 & ; \text{ elsewhere.} \end{cases}$$

Find  $E(Y|X = x)$  and  $\text{Var}(Y|X = x)$ .

(b) Define the characteristic function of a random variable. Show that the characteristic function of the sum of two independent random variables is equal to the product of their characteristic functions. Is converse true always ?

3. (a) The joint probability density function of a two-dimensional non-negative continuous random variable  $(X, Y)$  is given by

$$f(x, y) = \begin{cases} \alpha^{-2} e^{-\left(\frac{x+y}{\alpha}\right)}; & x > 0, y > 0, \alpha > 0 \\ 0 & ; \text{ otherwise.} \end{cases}$$

Find distribution of  $\frac{1}{2}(X - Y)$ .

(b) Compute  $R_{1.23}^2, r_{13.2}^2, \sigma_{1.23}^2$  and  $\text{Var}(e_{1.23})$ , when  $r_{12} = r_{13} = r_{23} = \rho$ .

4. (a) Let  $X$  and  $Y$  be two random variables with correlation coefficient ' $r$ ' and variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. If  $U = X + kY$  and  $V = X + \left(\frac{\sigma_1}{\sigma_2}\right)Y$ , find the value of ' $k$ ' so that  $U$  and  $V$  are uncorrelated.
- (b) Find the density function  $f(x)$  corresponding to the characteristic function :

$$\phi(t) = \begin{cases} 1 - |t|; & |t| \leq 1 \\ 0; & |t| > 1. \end{cases}$$

### Section-B

5. (a) Find the mean deviation about mean for Poisson ( $\lambda$ ) distribution. Also, compute mean deviation about mean for  $\lambda = 3.1415$ .
- (b) If  $X$  is a binomial variate with parameters  $n$  and  $p$ , prove that :

$$k_{r+1} = pq \frac{d}{dp} k_r; r = 1, 2, 3, \dots,$$

where  $k_r$  is the  $r$ th cumulant. Hence, find variance of  $X$ .

6. (a) If two i.i.d. random variables  $X$  and  $Y$  assume  $-1, 0$  and  $1$  with equal probabilities, then find m.g.f. of  $X + Y$  and hence write distribution of  $X + Y$ .
- (b) If the distribution of  $X$  is  $U(-5, 5)$ , then find characteristic function of  $10 + X$  and hence, find distribution of  $Y = 10 + X$ .

7. (a) Let  $X$  represents number of heads obtained in 10 tosses of a biased coin with probability of obtaining head  $\frac{1}{3}$  and the same coin tossed again  $X$  times and  $Y$  heads are obtained. Find  $\text{Var}(Y)$  and  $\text{Cov}(X, Y)$ .
- (b) Show that for  $N(\mu, \sigma^2)$  distribution  $\mu_{2r+1} = 0$  and  $\mu_{2r} = 1 \times 3 \times 5 \times \dots \times (2r-1)\sigma^{2r}$ .
8. (a) Find m.g.f. of normal distribution and use it to prove additive property of normal distribution.
- (b) If  $X$  and  $Y$  are two i.i.d. random variables with common probability density function

$$f(z) = \begin{cases} 2ze^{-z^2}; & z \geq 0 \\ 0 & ; \text{ otherwise.} \end{cases}$$

Find the distribution of  $\sqrt{X^2 + Y^2}$ .

