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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **5788**

Unique Paper Code : **2372011201**

Name of the Paper : **Theory of Probability Distributions (NEP)**

Name of the Course : **B.Sc. (Hons.) Statistics (DSC)**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting *three* questions from each section.

All questions/parts are of equal marks.

The symbols used have their usual meaning.

Use of non-programmable scientific calculator is allowed.

Section-A

1. (*a*) Suppose a two-dimensional continuous random variable (X, Y) has joint p.d.f. given by

$$f(x, y) = \begin{cases} 6x^2y; & 0 < x < 1, 0 < y < 1 \\ 0 \quad ; & \text{otherwise.} \end{cases}$$

P.T.O.

Find :

$$P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right),$$

$$P(X + Y < 1) \text{ and } P(X < 1 \mid Y < 2).$$

(*b*) Define the cumulant generating function and show that :

$$k_1 = \mu_1', k_2 = \mu_2, k_3 = \mu_3 \text{ and } k_4 = \mu_4 + 3\mu_2^2.$$

2. (*a*) Let

$$f(x, y) = \begin{cases} 8xy; & 0 < x < y < 1 \\ 0 \ ; & \text{elsewhere.} \end{cases}$$

Find $E(Y \mid X = x)$ and $Var(Y \mid X = x)$.

(*b*) Define the characteristic function of a random variable. Show that the characteristic function of the sum of two independent random variables is equal to the product of their characteristic functions. Is converse true always ?

3. (*a*) The joint probability density function of a two-dimensional non-negative continuous random variable (X, Y) is given by

$$f(x, y) = \begin{cases} \alpha^{-2} e^{-\left(\frac{x+y}{\alpha}\right)} ; & x > 0, y > 0, \alpha > 0 \\ 0 \ ; & \text{otherwise.} \end{cases}$$

Find distribution of $\frac{1}{2}(X - Y)$.

(*b*) Compute $R_{1.23}^2$, $r_{13.2}^2$, $\sigma_{1.23}^2$ and $Var(e_{1.23})$, when $r_{12} = r_{13} = r_{23} = \rho$.

4. (*a*) Let X and Y be two random variables with correlation coefficient '*r*' and variances σ_1^2 and σ_2^2, respectively. If U = X + *k*Y and $V = X + \left(\frac{\sigma_1}{\sigma_2}\right)Y$, find the value of '*k*' so that U and V are uncorrelated.

(*b*) Find the density function *f*(*x*) corresponding to the characteristic function :

$$\phi(t) = \begin{cases} 1 - |t|; & |t| \le 1 \\ 0; & |t| > 1. \end{cases}$$

Section-B

5. (*a*) Find the mean deviation about mean for Poisson (λ) distribution. Also, compute mean deviation about mean for λ = 3.1415.

(*b*) If X is a binomial variate with parameters *n* and *p*, prove that :

$$k_{r+1} = pq \frac{d}{dp} k_r; \; r = 1, 2, 3, \ldots\ldots,$$

where k_r is the *r*th cumulant. Hence, find variance of X.

6. (*a*) If two i.i.d. random variables X and Y assume –1, 0 and 1 with equal probabilities, then find m.g.f. of X + Y and hence write distribution of X + Y.

(*b*) If the distribution of X is U(–5, 5), then find characteristic function of 10 + X and hence, find distribution of Y = 10 + X.

7. (*a*) Let X represents number of heads obtained in 10 tosses of a biased coin with probability of obtaining head $\frac{1}{3}$ and the same coin tossed again X times and Y heads are obtained. Find Var(Y) and Cov(X, Y).

(*b*) Show that for $N(\mu, \sigma^2)$ distribution $\mu_{2r+1} = 0$ and $\mu_{2r} = 1 \times 3 \times 5 \times .... \times (2r - 1)\sigma^{2r}$.

8. (*a*) Find m.g.f. of normal distribution and use it to prove additive property of normal distribution.

(*b*) If X and Y are two i.i.d. random variables with common probability density function

$$f(z) = \begin{cases} 2ze^{-z^2}; & z \geq 0 \\ 0 \quad ; & \text{otherwise.} \end{cases}$$

Find the distribution of $\sqrt{X^2 + Y^2}$.