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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **5579**

Unique Paper Code : **2372013603**

Name of the Paper : **Econometrics (DSC-NEP)**

Name of the Course : **B.Sc.(H) Statistics**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question no. **1** is compulsory.

Attempt *five* more questions choosing at least **2** from each section.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Fill in the blanks :

(*i*) For $\underset{\sim}{Y} = X\underset{\sim}{\beta} + \underset{\sim}{u}$, where $E(\underset{\sim}{u}\underset{\sim}{u}') = V$, the Generalized least squares estimator is given by ....

(*ii*) For GLM, we can express $\underset{\sim}{e} = M\underset{\sim}{u}$, where matrix M = ...

(*iii*) If the Durbin-Watson *d* test statistic is found to be equal to 0, then .... order autocorrelation is ....

(*iv*) Spearman's rank correlation test is used to detect ....

(*v*) If there exists high multicollinearity, then the regression coefficients are ....

(*b*) State whether True/False. If Flase, then give the correct statement.

(*i*) Farrar Glauber test offers a solution to the problem of Heteroscedasticity.

P.T.O.

(*ii*) Multicollinearity is a violation of assumption pertaining to error terms.

(*iii*) The ordinary least squares procedure yields the BLUE of parameters of a General Linear model.

(*iv*) Aitken estimator is used when the X matrix is not a full rank matrix.

(*v*) A hypothesis such as $H_0 : \beta_2 = \beta_3 = 0$ can be tested using the t test.

(*c*) (*i*) Consider the data :

Y	–4	–2	0	2	4
X_2	1	2	3	4	5
X_3	5	7	9	11	13

Can the parameters of the model, $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ be estimated ?

Why or why not, if not then what linear functions of these parameters can you estimate ? Show the necessary calculations.

(*ii*) From a cross-sectional data on 59 countries, the following regression was obtained :

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

Running the auxiliary regression

$$e_1^2 = -\ 5.8417 + 2.5629\ X_{2i} + 0.6918\ X_{3i} - 0.4081(X_{2i})^2 - 0.049\ (X_{3i})^2 + 0.0015(X_{2i})(X_{3i});$$

with $R^2 = 0.1148$. Test the presence of heteroscedasticity using White's Heteroscedasticity test. (given that the chi-square value at 5% level of significance is 11.0705). $5,5,2\frac{1}{2},2\frac{1}{2}$

Section-A

2. (*i*) Consider the following *two* models for n sample observations :

$$\text{Model I} : Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$\text{Model II} : Y_t = \alpha_1 + \alpha_2 (X_t - \overline{X}) + u_t$$

Obtain the expressions of OLS estimators of β_1, β_2, α_1 and α_2. Show that OLS estimators of β_1 and α_1 are not identical, however, OLS estimators of β_2 and α_2 are identical. Also, obtain expressions for variances of OLS estimators of β_1, β_2, α_1 and α_2.

(*ii*) For General Linear model, obtain $100(1 - \alpha)\%$ confidence interval for $E\left[Y_{n+1} \mid \underset{\sim}{c}\right]$ where $\underset{\sim}{c}' = \left(1, X_{2,n+1}, \ldots\ldots, X_{k,n+1}\right)$. Also obtain $100(1 - \alpha)\%$ confidence interval for the individual value Y_{n+1}. 7,8

3. (*i*) Obtain the estimates of the coefficients of the linear relation.

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

subject to the linear restriction $\beta_2 = \beta_3$. It is given that $n = 25$, $X'X = \begin{bmatrix} 20 & 0 \\ 0 & 40 \end{bmatrix}$ and $X'Y = \begin{bmatrix} 15 \\ 25 \end{bmatrix}$.

(*ii*) Describe the terms "perfect Multicollinearity" and "high-but-imperfect Multicollinearity". Discuss the consequences when OLS formulae are applied to both of the cases ? Illustrate your answer with the help of a suitable example. 7,8

4. (*i*) Discuss the solutions of Multicollinearity.

(*ii*) Discuss the method based on Frisch Confluence Analysis to detect the presence of Multicollinearity. 7,8

P.T.O.

Section-B

5. (i) Explain the Cochrane-Orcutt iterative procedure for a 2-variable regression model where disturbances follow first order autoregressive scheme.

(ii) For the *two* variable regression model, $Y_t = \beta_1 + \beta_2 X_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ with $E(\epsilon_t) = 0$, $v(\epsilon_t) = \sigma_g^2$ and $\text{cov}(\epsilon_t, \epsilon_s) = 0$, for $t \neq s$, derive the expression for the mean and variance-covariance matrix of $\underset{\sim}{u}$, where

$\underset{\sim}{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$. Further, under this setup, obtain the expression for the variance of OLS estimator $\hat{\beta}_2$. How does it compare with the OLS formulae of variance of $\hat{\beta}_2$? 7,8

6. (i) Discuss the test based on V on Neumann ratio for the detection of autocorrelation. If the value of the Durbin Watson test statistic is $d = 1.875$ based on 28 sample observations, then obtain the value of Von Neumann ratio.

(ii) In a two variables regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$, where $E(u_i) = 0$, $V(u_i) = \sigma^2 X_i^2$ for all $i = 1, 2, \ldots.. n$, derive the expression of the Generalized least square estimator of β_2 along with its variance. Further obtain the variance of OLS estimator of β_2 under heteroscedasticity. Comment on the performance of the estimators by using the values of $X_i = 1, 2, 3, 4, 5$. 7,8

7. (i) Discuss the Goldfeld-Quandt test for heteroscedasticity. How do you proceed with the test when there is more than one variable in the model ?

(ii) For the General Linear Model, $\underset{\sim}{Y} = X\beta + \underset{\sim}{u}$, where $E(\underset{\sim}{u}) = \underset{\sim}{0}$ and $E(\underset{\sim}{u}\underset{\sim}{u}') = V$, where V is a known symmetric positive definite matrix. Find the best linear unbiased predictor of a single value of the regressand y_0, given the row vector of prediction regressors $\underset{\sim}{x}_0$. 7,8