

This question paper contains 4 printed pages]

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S. No. of Question Paper : **5579**

Unique Paper Code : **2372013603**

Name of the Paper : **Econometrics (DSC-NEP)**

Name of the Course : **B.Sc.(H) Statistics**

Semester : **VI**

Duration : **3 Hours**

Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question no. 1 is compulsory.

Attempt *five* more questions choosing at least **2** from each section.

Use of non-programmable scientific calculator is allowed.

1. (a) Fill in the blanks :

(i) For $\underline{Y} = X\underline{\beta} + \underline{u}$, where $E(\underline{u}\underline{u}') = V$, the Generalized least squares estimator is given by

(ii) For GLM, we can express $\underline{e} = M\underline{u}$, where matrix $M = \dots\dots\dots$

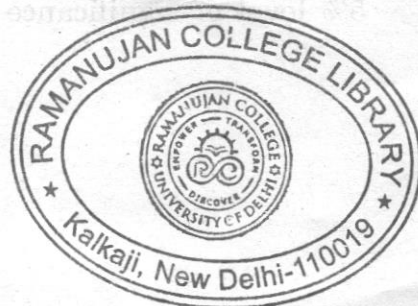
(iii) If the Durbin-Watson d test statistic is found to be equal to 0, then order autocorrelation is

(iv) Spearman's rank correlation test is used to detect

(v) If there exists high multicollinearity, then the regression coefficients are

(b) State whether True/False. If False, then give the correct statement.

(i) Farrar Glauber test offers a solution to the problem of Heteroscedasticity.



P.T.O.

- (ii) Multicollinearity is a violation of assumption pertaining to error terms.
- (iii) The ordinary least squares procedure yields the BLUE of parameters of a General Linear model.
- (iv) Aitken estimator is used when the X matrix is not a full rank matrix.
- (v) A hypothesis such as $H_0 : \beta_2 = \beta_3 = 0$ can be tested using the t test.
- (c) (i) Consider the data :

Y	-4	-2	0	2	4
X ₂	1	2	3	4	5
X ₃	5	7	9	11	13

Can the parameters of the model, $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ be estimated ?

Why or why not, if not then what linear functions of these parameters can you estimate ? Show the necessary calculations.

- (ii) From a cross-sectional data on 59 countries, the following regression was obtained :

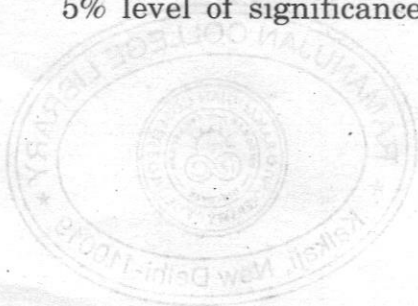
$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

Running the auxiliary regression

$$e_1^2 = -5.8417 + 2.5629 X_{2i} + 0.6918 X_{3i} - 0.4081(X_{2i})^2 - 0.049 (X_{3i})^2 + 0.0015(X_{2i})(X_{3i});$$

with $R^2 = 0.1148$. Test the presence of heteroscedasticity using White's Heteroscedasticity test. (given that the chi-square value at 5% level of significance is 11.0705).

$$5, 5, 2\frac{1}{2}, 2\frac{1}{2}$$



Section-A

2. (i) Consider the following *two* models for n sample observations :

$$\text{Model I : } Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$\text{Model II : } Y_t = \alpha_1 + \alpha_2 (X_t - \bar{X}) + u_t$$

Obtain the expressions of OLS estimators of β_1 , β_2 , α_1 and α_2 . Show that OLS estimators of β_1 and α_1 are not identical, however, OLS estimators of β_2 and α_2 are identical. Also, obtain expressions for variances of OLS estimators of β_1 , β_2 , α_1 and α_2 .

- (ii) For General Linear model, obtain $100(1 - \alpha)\%$ confidence interval for $E [Y_{n+1} | c]$ where $c' = (1, X_{2,n+1}, \dots, X_{k,n+1})$. Also obtain $100(1 - \alpha)\%$ confidence interval for the individual value Y_{n+1} . 7,8

3. (i) Obtain the estimates of the coefficients of the linear relation.

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

subject to the linear restriction $\beta_2 = \beta_3$. It is given that $n = 25$, $X'X =$

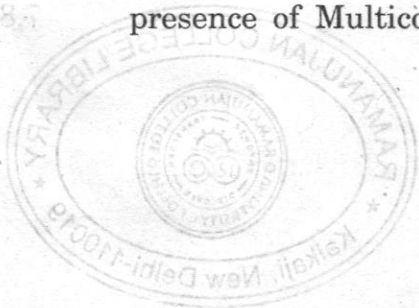
$$= \begin{bmatrix} 20 & 0 \\ 0 & 40 \end{bmatrix} \text{ and } X'Y = \begin{bmatrix} 15 \\ 25 \end{bmatrix}.$$

- (ii) Describe the terms "perfect Multicollinearity" and "high-but-imperfect Multicollinearity". Discuss the consequences when OLS formulae are applied to both of the cases ? Illustrate your answer with the help of a suitable example. 7,8

4. (i) Discuss the solutions of Multicollinearity.

- (ii) Discuss the method based on Frisch Confluence Analysis to detect the presence of Multicollinearity. 7,8

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Section-B

5. (i) Explain the Cochrane-Orcutt iterative procedure for a 2-variable regression model where disturbances follow first order autoregressive scheme.

- (ii) For the two variable regression model, $Y_t = \beta_1 + \beta_2 X_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ with $E(\epsilon_t) = 0$, $V(\epsilon_t) = \sigma_\epsilon^2$ and $\text{cov}(\epsilon_t, \epsilon_s) = 0$, for $t \neq s$, derive the expression for the mean and variance-covariance matrix of \underline{u} , where

$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$. Further, under this setup, obtain the expression for the variance of OLS estimator $\hat{\beta}_2$. How does it compare with the OLS formulae of variance of $\hat{\beta}_2$? 7,8

6. (i) Discuss the test based on V on Neumann ratio for the detection of autocorrelation. If the value of the Durbin Watson test statistic is $d = 1.875$ based on 28 sample observations, then obtain the value of Von Neumann ratio.

- (ii) In a two variables regression model $Y_i = \beta_1 + \beta_2 X_i + u_i$, where $E(u_i) = 0$, $V(u_i) = \sigma^2 X_i^2$ for all $i = 1, 2, \dots, n$, derive the expression of the Generalized least square estimator of β_2 along with its variance. Further obtain the variance of OLS estimator of β_2 under heteroscedasticity. Comment on the performance of the estimators by using the values of $X_i = 1, 2, 3, 4, 5$. 7,8

7. (i) Discuss the Goldfeld-Quandt test for heteroscedasticity. How do you proceed with the test when there is more than one variable in the model ?

- (ii) For the General Linear Model, $\underline{Y} = \underline{X}\underline{\beta} + \underline{u}$, where $E(\underline{u}) = \underline{0}$ and $E(\underline{u}\underline{u}') = \underline{V}$, where \underline{V} is a known symmetric positive definite matrix. Find the best linear unbiased predictor of a single value of the regressand y_0 , given the row vector of prediction regressors \underline{x}_0 . 7,8

