(b) The length of the skulls of 10 fossil skeletons of an extinct species of bird has a mean of 5.68 cm and a standard deviation of 0.29 cm. Assuming that such measurements are normally distributed, find a 95% confidence interval for the mean length of the skulls of this species of bird. (Given that $Z_{0.025} = 1.96$, $t_{0.025,9}$ 2.262.) (9,6)

7. (a) What is a statistical hypothesis? Define the terms: (i) level of significance, (ii) best critical region, and (iii) null and alternative hypotheses.

(b) Given the frequency function:

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & elsewhere \end{cases}$$

and that you are testing the hypothesis H_0: $\theta = 1$, against H_1: $\theta = 2$ by means of a single observed value of x. What would be the sizes of the type I and type II errors, if the interval $x \ge 0.5$ is the critical region? Also obtain power of the test. (7, 8)

8. Write short notes on any three:

(i) Two types of errors

(ii) UMP test

(iii) UMPU test

(iv) Method of least square (15)

(500)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4868 **J**

Unique Paper Code : 2374002004

Name of the Paper : Basics of Statistical Inference

Name of the Course : **Statistics: Generic Elective under NEP-UGCF**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt **Six** questions in all selecting Three from each section.

3. Attempt **all** parts of question in continuation.

4. Use of simple calculator is allowed.

Section I

1. (a) What do you understand by point estimation? When would say that estimator of a parameter is good?

In particular, discuss the requirements of consistency and efficiency of an estimator with illustrations.

(b) Show that $s^2 = \frac{1}{n}\sum(X_i - \bar{X})^2$ is a biased estimator of σ^2, where $\bar{X}$ is the mean of the random sample $X_1, X_2, \ldots, X_n$. Also, show that $S^2 = \frac{n}{n-1}s^2$ is an unbiased estimator of σ^2.

(9,6)

2. Define Minimum variance unbiased estimator (MVUE) and Minimum variance bound estimator (MVB) and explain clearly the difference between them. Given a random sample of size n from a population with p.d.f.:

$$f(x,\theta) = \frac{1}{\theta}e^{\frac{-x}{\theta}};\ x > 0,\ \theta > 0$$

Show that the mean is the MVB estimator of the parameter θ. (15)

3. (a) Discuss the terms:

(i) estimate,

(ii) sufficient statistic, and

(iii) completeness of a statistic.

(b) Explain Fisher-Neyman criterion. A random sample $X_1, X_2, \ldots, X_n$ is drawn from a Normal population $N(\mu, \sigma^2)$. Find sufficient estimators for μ and σ^2.

(6,9)

4. Write short notes on any three:

(i) Unbiased estimator

(ii) Cramer-Rao inequality

(iii) Rao-Blackwell Theorem

(iv) Lehmann-Scheffe Theorem (15)

Section II

5. (a) In random sampling from a Normal population $N(\mu, \sigma^2)$, find the MLES for

(i) μ, when σ^2 is known,

(ii) σ^2, when μ is known, and

(iii) the simultaneous estimation of μ and σ^2.

(b) Show by means of an example, that MLEs are not necessarily unbiased. (9, 6)

6. (a) Explain the method of constructing $100(1-\alpha)\%$ confidence interval for μ of Normal population with mean μ and variance σ^2. How do you proceed if σ^2 is (i) known, and (ii) unknown.