Section-III

7. (a) Obtain the moment generating function of Logistic distribution and hence find its mean and variance.

(b) Let X follows a standard Cauchy distribution. Find the p.d.f for X^2 and identify its distribution. (7,8)

8. (a) If X and Y are independent with common p.d.f. (exponential) :

$$f(x)=\begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the p.d.f of X – Y and identify its distribution.

(b) Find the characteristic function of standard Laplace distribution and hence find its mean and standard deviation. (7,8)

9. (a) If $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ are independent random variables, then find the distribution of $\frac{X}{Y}$ and identify it.

(b) If $X_1, X_2, X_3, \ldots, X_n$ are independent random variables and X_i has an exponential distribution with parameter θ_i, i = 1,2, ..., n; then show that $Z = \min(X_1, X_2, X_3, \ldots, X_n)$ has an exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$ (i.e. $n\theta$). (7,8)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1144 I

Unique Paper Code : 2372012302

Name of the Paper : Advanced Probability Distributions, DSC–8

Name of the Course : **B.Sc. (Hons.) Statistics (Under NEP-UGCF)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **two** questions from each section.
3. **All** questions carry equal marks.

Section-I

1. (a) Derive moment generating function of Negative Binomial Distribution. Obtain its mean and variance from M.G.F. and hence show that mean is less than variance.

(b) Let X_1 and X_2 be independent r.v.'s each having Geometric distribution $q^k p$; $k = 0,1,2,...$ Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform. (7,8)

2. (a) Suppose X is a non-negative integral valued random variable. Show that the distribution of X is geometric if it "lacks memory", i.e., if for each $k \geq 0$ and $Y = X - K$, one has $P(Y = t \mid X \geq K) = P(X = t)$, for $t \geq 0$.

(b) If $X_1, X_2, ..., X_k$ are k independent Poisson variates with parameters $\lambda_1, \lambda_2 ... \lambda_k$ then prove that the conditional distribution

$P(X_1 \cap X_2 \cap ... \cap X_k \mid X)$, where $X = X_1 + X_2 + \cdots + X_k$

is multinomial. (7,8)

3. (a) A box contains N items of which 'a' items are defective and 'b' are non-defective, $(a + b = N)$. A sample of 'n' items is drawn at random. Let X be the number of defective items in the sample. Obtain the probability distribution of X. And show that the Binomial distribution is a limiting case of Hyper-geometric distribution.

(b) Obtain the first four cumulants of negative binomial distribution. And hence find β_1 and β_2. (7,8)

Section-II

4. (a) If X & Y are independent Gamma variates with parameters μ and ν respectively, show that $U = X + Y$, $Z = \dfrac{X}{Y}$ are independent and U is a $\gamma(\mu + \nu)$ variate and Z is a β_2 (μ, ν) variate.

(b) Define Beta distribution of First kind. Compute r^{th} moment and harmonic mean for Beta distribution of First kind. (7,8)

5. (a) If X has a uniform distribution in [0,1]. Find the pdf of -2logX and identify the distribution.

(b) If X is a gamma variate with single parameter λ, then obtain its m.g.f. Hence deduce that the m.g.f. of standard gamma variate tends to $\exp\left(\dfrac{t^2}{2}\right)$ as $\lambda \to \infty$. (7,8)

6. (a) Define rectangular (or uniform) distribution and find its mean, variance and mean deviation about mean.

(b) Show that mean value of positive square root of a $\gamma(\lambda, n)$ variate is $\gamma\left(n + \dfrac{1}{2}\right) \Big/ \left(\sqrt{\lambda}, \gamma(n)\right)$. Hence prove that mean deviation of a $N(\mu, \sigma^2)$ variate from its mean is $\sqrt{2/\pi}$. (7,8)