This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5828**

Unique Paper Code : **2372011203**

Name of the Paper : **Algebra for Statistics**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **II (NEP)**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting *three* questions from each Section.

All questions/parts carry equal marks.

Use of non-programmable scientific calculators is allowed.

SECTION A

1. (*a*) Define idempotent, involutory and trace of a matrix. Prove that :

   (*i*) $tr(\mathrm{ABC}) = tr(\mathrm{BCA}) = tr(\mathrm{CAB})$

   (*ii*) $tr(\mathrm{C}^{-1}\ \mathrm{AC}) = tr(\mathrm{A})$.

   (*b*) If A is a non-singular matrix of order n, then show that :

   (*i*) $|\mathrm{adj\,A}| = |\mathrm{A}|^{n-1}$

   (*ii*) $|\mathrm{adj(adj\,A)}| = |\mathrm{A}|^{(n-1)^2}$

   (*iii*) $(\mathrm{adj\,A}) = |\mathrm{A}|^{n-2} \cdot \mathrm{A}$.

P.T.O.

2. (*a*) A mixture of weight 6 kilograms is to be made containing foods A, B and C. It is given that A, B, C contains 500, 300, 200 units of vitamins and 400, 600, 700 units of calories per kg. The mixture is to contain 1800 units of vitamins and 3500 units of calories. Using matrices, find the composition of the mixture by forming the necessary simultaneous linear equations.

(*b*) When is a rectangular matrix said to be in Echelon form ? Transform the following matrix into an echelon form and encircle the pivot entries :

$$\begin{pmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{pmatrix}.$$

3. (*a*) Define Circulant Determinant. Prove that :

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c).$$

(*b*) Show that every square matrix can be uniquely expressed as P + *i*Q where P and Q are Hermitian matrices.

4. (*a*) Compute the inverse of the given matrix by the method of partitioning :

$$A = \begin{pmatrix} I & P \\ Q & R \end{pmatrix}.$$

where I is an identity matrix, P, Q, R are sub-matrices and it is given that the necessary inverses exist.

(*b*) Express the determinant :

$$\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$$

as a product of two determinants. Hence or otherwise show that it vanishes.

SECTION B

5. (*a*) Define Vector spaces, Lincar Dependence and Independence of vectors. Show that the vectors [1, 2, 4], [2, 2, 8], [1, 0, 4] are linearly dependent.

(*b*) Show that any two characteristic vectors corresponding to two distinct characteristic roots are orthogonal for the following matrices :

(*i*) Hermitian matrix,

(*ii*) Real symmetric matrix.

(*iii*) Unitary matrix.

6. (*a*) Reduce the following symmetric matrix to the diagonal form and interpret the result in terms of quadratic forms :

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{pmatrix}.$$

(*b*) Find non-singular matrices R and S such that RAS is in normal form, where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}.$$

Hence find the rank of A.

7. (*a*) Define generalized inverse. Give an algorithm for finding a generalized inverse of a symmetric matrix A. Hence find a *g*-inverse of

$$\begin{pmatrix} 2 & 2 & 6 \\ 2 & 3 & 8 \\ 6 & 8 & 22 \end{pmatrix}.$$

(*b*) The characteristic roots of a square matrix of order 3 are 1, –1, 2. Express A^6 as the quadratic polynomial in A.

8. (*a*) Show that elementary matrices are non-singular. Obtain their inverses.

(*b*) Show that the form :

$$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$$

is indefinite and find sets of values of the variables x_1, x_2, x_3 for which the given form assumes positive, negative and zero values.