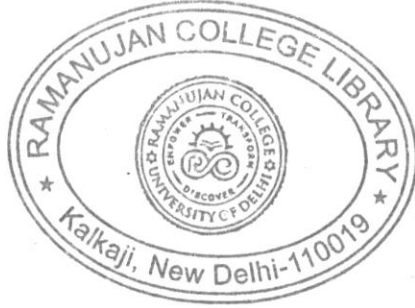


5539

8

Point out the difference between one-tail and two-tail tests. Briefly explain how a statistical hypothesis is tested. [8,7]



(700)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5539 J

Unique Paper Code : 2372012401

Name of the Paper : Sampling Distributions

Name of the Course : B.Sc. (Hons.), Statistics (NEP-UGCF)

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all by selecting **three** questions from each section.
3. All questions carry equal marks.
4. Use of simple calculator is allowed.

P.T.O.

SECTION I

1. (a) If X is a chi-square variate with n d.f., then prove that for large n , $\sqrt{2X} \sim N(\sqrt{2n}, 1)$
- (b) Let t has Student's t -distribution with 2 d.f.. Find the probability $P[t \geq 2]$.
- (c) X is a F -variate with 2 and n ($n \geq 2$) d.f.. Find the probability $p = P[F \geq k]$ and deduce the significance level of F corresponding to the significance level of probability p .
- [5, 5, 5]
2. (a) Let X_1, X_2, \dots, X_n , be a random sample from $N(\mu, \sigma^2)$, and k be a positive integer. Find $E[S^{2k}]$. In particular, find $E[S^2]$ and $V[S^2]$.
- (b) Let X_1, X_2, \dots, X_n be a random sample from $N(\mu, \sigma^2)$.

$n\lambda$ if and only if each X_i is exponential with parameter λ . [8,7]

8. (a) Explain the term Sampling distribution and standard error of a statistic. Derive the expression for the standard error of
- (i) the sample mean of a random sample of size n from a population having finite population variance σ^2
- (ii) the difference of means of two independent random samples of size n_1 and n_2 from two populations having finite variances of σ_1^2 and σ_2^2 respectively.
- (b) What are the two types of errors that arise in testing of hypothesis?

(b) Define convergence in law, convergence in probability and convergence with probability one. State the relationship between convergence in probability and convergence with probability one and prove it. [8,7]

7. (a) Let $X_1, X_1, \dots, X_{2n-1}$ be an odd -size random sample from a $N(\mu, \sigma^2)$ population.

Find p.d.f. of the sample median and show that it is symmetric about μ and has the mean μ .

- (b) Let X_1, X_1, \dots, X_n be a random sample from a population with continuous density. Show that $Y_1 = \text{Min}(X_1, X_2, \dots, X_n)$ is exponential with parameter

Let \bar{X} and S^2 be the sample mean and sum of squares of the deviations from the mean respectively. Let X' be one more observation independent of previous ones. Find the Sampling distribution of

$$U = \frac{X' - \bar{X}}{S} \left[\frac{n(n-1)}{n+1} \right]^{\frac{1}{2}}$$

[8, 7]

3. (a) Define Student's t-statistic and Fisher's t-statistic. Show that Student's t-statistic may be regarded as a particular case of Fisher's t-statistic. Obtain the p.d.f. of Student's t-statistic.

- (b) Find the p.d.f. of $\chi_n = +\sqrt{\chi_n^2}$ and $\mu_r = E[\chi_n^r]$, where χ_n^2 is a χ^2 -variate with n d.f.

Hence, establish that for large n ,

$$E[\chi_n^2] = [E(\chi_n)]^2$$

[8, 7]

4. (a) Let $X \sim F_{m,n}$. Find the mean, and mode of X . Also, find the distribution of

$$U = \frac{mX}{n+mX}$$

- (b) Show that a χ^2 -test involving two sample proportions is equivalent to a large sample significance test of difference in the proportions.

[8,7]

SECTION II

5. (a) State and prove Chebychev's Inequality. Use Chebychev's inequality to show that for $n > 36$, the

probability that in n throws of a fair die, the number

of sixes lies between $\frac{n}{6} - \sqrt{n}$ and $\frac{n}{6} + \sqrt{n}$ is

at least $31/36$.

- (b) Let $\{X_n\}$ be a sequence of mutually independent random variables such that: $X_n = \pm 1$ with probability $\frac{1-2^{-n}}{2}$ and $X_n = \pm 2^{-n}$ with probability 2^{-n-1} . Examine whether the W.L.L.N. holds for the sequence $\{X_n\}$. [8,7]

6. (a) Show that the central limit theorem holds for the sequence $\{X_k\}$ of independent random variables defined as $P[X_k = 0] = 1 - k^{1-2\alpha}$, $P[X_k = \pm k^\alpha] = \frac{1}{2}k^{-2\alpha}$ if $\alpha < \frac{1}{2}$.