This question paper contains 7 printed pages]

**Roll No.** | | | | | | | | | | |



S. No. of Question Paper : **5816**

Unique Paper Code : **2342011203**

Name of the Paper : **Probability for Computing**

Name of the Course : **B.Sc. (H) Computer Science**

Semester : **II / DSC**

Duration : **3 Hours** Maximum Marks : **90**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

The paper has *two* Sections A and B.

Section A is compulsory.

Attempt any *four* questions from Section B.

Each question is of **15** marks.

Answer all parts of a question together.

Use of scientific calculator is allowed.

Use of z table is allowed.

## Section A

1. (*a*) What will be the sample space for the following cases : 3

(*i*) If the experiment consists of measuring the lifetime of a car.

(*ii*) If the experiment consists of rolling the two dice and records the sum of the numbers shown.

(*iii*) If the experiment consists of recording the attendance of a class having 50 students.

(*b*) Suppose that the chance of rain tomorrow depends on previous weather conditions only through whether or not it is raining today and not on past weather conditions. Suppose also that if it rains today, then it will rain tomorrow with probability $\alpha$; and if it does not rain today, then it will rain tomorrow with probability $\beta$. Provide the transition matrix for the same. 3

(*c*) Prove that any events say E and E' are always mutually exclusive. Use a suitable example. 3

(*d*) Suppose X has a probability mass function given by 3

$$p(1) = \frac{1}{2},\ p(2) = \frac{1}{3},\ p(3) = \frac{1}{6}$$

Write down the cumulative distribution function F of X.

(*e*) What are the mathematical conditions to say that a state is : 3

(*i*) A recurrent state

(*ii*) A transient state.

(*f*) Suppose that the number of phone calls received by a call center in a 1-minute interval follows a Poisson distribution with parameter $\lambda = 2$. What is the probability that no calls are received in a given 1-minute interval ? 3

(*g*) Suppose that $p(x, y)$ the joint probability mass function of X and Y is given by

$p(1, 1) = 0.5$, $p(1, 2) = 0.1$, $p(2, 1) = 0.1$, $p(2, 2) = 0.3$.

Calculate the conditional probability mass function of X given that Y = 1. 3

(*h*) A coin, having probability $p$ of coming up heads, is to be successively flipped until the first head appears. What is the expected number of flips required ? 3

(*i*) Explain how are pseudo random numbers generated using a seed value. 3

(*j*) List *three* properties of covariance. 3

## Section B

2. (*a*) A family has two children. What is the conditional probability that both are boys given that at least one of them is a boy ? For instance (*b*, *g*) means that the older child is a boy and the younger child a girl. 5

(*b*) Calculate the cumulative distribution function of a random variable uniformly distributed over (α, β). 5

(*c*) Suppose the joint density of X and Y is given by

$$f(x) = \begin{cases} 12x^2 y, & 0 < x < 1, \ 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Compute the conditional expectation of X given that Y = $y$, where $0 < y < 1$. 5

3. (*a*) A Markov chain has the following transition matrix :

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

Let the states be $S_0$, $S_1$, $S_2$. Using the Chapman-Kolmogorov equation, compute the probability that the process is in state $S_0$ after 2 steps, given that it started in state $S_2$. 5

(*b*) Suppose X has the following probability mass function :

$$p(0) = 0.2,\ p(1) = 0.5,\ p(2) = 0.3.$$

Calculate $E[X^2]$. And check if $E[X^2] = E[X]^2$. 5

(*c*) The joint density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{2} y e^{-xy}, & 0 < x < \infty,\ 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

What is $E\left[e^{\frac{X}{2}} \middle| Y = 1\right]$ ? 5

4. (*a*) Calculate E[X], when X is binomially distributed with parameters $n$ and $p$. 5

(*b*) Let X be the number of times that a fair coin is flipped 40 times, lands heads. Find the probability that X = 20 using normal approximation. 5

(c) A company produces 70% of its products in *Location* "A" and 30% in *Location* "B". The fault rate is 2% for products in *Location* "A" and 5% for products in *Location* "B". If a randomly selected product is found to have a fault, calculate the probability of product being produced in *Location* "A" ? 5

5. (a) A set of **four cards numbered 1 through 4**, is contained in a bag. One card is randomly selected. Given the following events :

$$X = \{1, 3\}, Y = \{1, 2\}, Z = \{1, 4\}.$$

Check if these events X, Y and Z are jointly independent or not. 5

(b) Let X be a random variable with finite mean $\mu$ and variance $\sigma^2$. Using **Markov's Inequality**, prove that for any $k > 0$,

$$P\{|X - \mu| \geq\} k \leq \sigma^2/k^2.$$

Also, name the inequality obtained. 5

(c) Suppose a Markov chain contains states 0, 1, 2 and 3 and

$$p = \begin{vmatrix} 0 & 0 & .5 & .5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{vmatrix}$$

Find which state is recurrent. 5

6. (*a*) Rajit will read either one chapter of his probability book or one chapter of his history book. If the number of misprints in a chapter of his probability book is Poisson distributed with mean 3 and if the number of misprints in his history chapter is Poisson distributed with mean 6, then assuming Rajit is equally likely to choose either book, what is the expected number of misprints that Rajit will come across ? 5

(*b*) Consider a weather model where if it rains today (state 0), it will rain tomorrow with probability $\alpha$; and if it does not rain today (state 1), it will rain tomorrow with probability $\beta$. If we say that the state is 0 when it rains and 1 when it does not rain, then find the limiting probabilities $\pi_0$ and $\pi_1$. 5

(*c*) If X and Y are independent, then for any functions $h$ and $g$, prove that : 5

$$E[g(X)h(Y)] = E[g(X)]E[h(Y)].$$

7. (*a*) Suppose there are 25 different types of coupons and suppose that each time one obtains a coupon, it is equally likely to be any one of the 25 types. Compute the expected number of different types that are contained in a set of 10 coupons. 5

(*b*) Suppose the number of customers arriving at a store in a day is a random variable with a mean of 200. 5

(*i*) What can be said about the probability that today's customer count will be at least 300 ?

(*ii*) If the variance of the daily customer count is known to be 50, what can be said about the probability that the customer count will be between 180 and 220 ?

(*c*) In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $p$ be the probability that the student knows the answer, and $1 - p$ be the probability that the student guesses. If the student guesses, the probability of answering correctly is $\frac{1}{4}$, where 4 is the number of multiple-choice alternatives.

What is the conditional probability that the student knew the answer, given that she answered the question correctly ? 5

(ii) If the variance of the daily customer count is known to be 50, what can be said about the probability that the customer count will be between 180 and 220 ?

(c) In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let $p$ be the probability that the student knows the answer, and $1 - p$ be the probability that the student guesses. If the student guesses, the probability of answering correctly is $\frac{1}{4}$, where 4 is the number of multiple-choice alternatives. What is the conditional probability that the student knew the answer given that she answered the question correctly ? 5