18/6/25 (M)

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 5823

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II/DSC

Duration : 3 Hours Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any *two* parts from each question.

Each part carries **7.5** marks.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Solve the initial value problem :

$$(2y \sin x \cos x + y^2 \sin x)\, dx + (\sin^2 x - 2y \cos x)\, dy = 0,\ y(0) = 3.$$

(*b*) Solve :

$$(x^2 - 3y^2)\, dx + 2xy\, dy = 0.$$

(*c*) Solve :

$$x^2 y'' = 2xy' + (y')^2$$

by reduction of order.

2. (*a*) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants ?

P.T.O.

(b) A metal bar at a temperature of 100°F is placed in a room at a constant temperature of 0°F. If after 20 minutes the temperature of the bar is half, find an expression for the temperature of the bar at any time.

(c) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei in the sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 200 years. What percentage of the original radioactive nuclei will remain after 1000 years ?

3. (a) Show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$$

Find the particular solution satisfying the initial condition,

$$y(0) = 0, \; y'(0) = 0.$$

(b) Solve the differential equation using the method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$

(c) Find the general solution of the differential equation using the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x.$$

4. (a) Use operator method to find the general solution of the following linear system :

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dy}{dt} + 2x + y = 3e^t.$$

(*b*) Solve the initial value problem, assuming $x > 0$

$$x^3 \frac{d^3y}{dx^3} - x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3.$$

(*c*) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ m and initial velocity $v_0 = -10$ m/s. Find the amplitude, period and frequency of the resulting motion.

5. (*a*) Define carrying capacity. Derive the logistic equation :

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right)$$

where r and K are reproduction rate and carrying capacity respectively.

(*b*) Suppose a soft drink manufacturing company has a 1000 litre tank containing sugar water solution. Initially s_0 kg of sugar is dissolved. Sugar solution flows into the tank at the rate of 10 litre/min with concentration $cin(t)$ kg/l. Assume that the solution in the tank is thoroughly mixed and the sugar solution flows out at the same rate at which it flows in i.e. the volume of sugar water mixture in the tank remains constant. Find a differential equation for the amount of sugar in the tank at any time t. Also solve it.

(*c*) Suppose a population can be modelled using the differential equation :

$$\frac{dx}{dt} = 0.2x - 0.001x^2$$

with the initial population size of $x_0 = 100$ at a time step of 1. Find the predicted population after two months.

6. (*a*) The given differential equation :

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{N_T}\right)$$

represents the model for spread of technology where N(*t*) is the number of ranchers who adopted an improved pasture technology in Uruguay and N_T is the total population of ranchers. It is assumed that rate of adoption is proportional to both the number who have adopted the technology and the fraction of population of ranchers who have not adopted the technology.

(*i*) Which term corresponds to the fraction of the population who have not yet adopted the improved pasture technology ?

(*ii*) According to the Banks (1994), $N_T = 17015$, $a = 0.490$ and $N_0 = 141$. Determine how long it takes for the improved pasture technology to spread to 80% of the population.

(*b*) Derive the epidemic model of influenza

$$\frac{dS}{dt} = -\beta SI, \frac{dI}{dt} = \beta SI - \gamma I, \frac{dS}{dt} = \gamma I$$

where β and γ are positive constants.

(*c*) Define equilibrium points. Find all equilibrium points of Predator-Prey model

$$\frac{dX}{dt} = \beta_1 X\left(1 - \frac{X}{K}\right) - c_1 XY, \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

where the positive constants c_1, c_2 are interaction parameter, α_2 is predator per capita death rate and β_1 is prey per capita birth rate. Discuss the direction of trajectories of Predator-Prey model.