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Roll No. | | | | | | | | | | |

S. No. of Question Paper : **5673**

Unique Paper Code : **2352012403**

Name of the Paper : **Numerical Analysis**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory. Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Define order of convergence of a sequence $\{p_n\}$ which converges to a number p. The sequence generated by the formula $x_{n+1} = \dfrac{x_n^3 + 3x_n a}{3x_n^2 + a}$ converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.

(*b*) Perform four iterations of Bisection method to approximate the root of the equation $x^4 - x - 10 = 0$, in the interval (1, 2).

(*c*) Explain geometrically the Newton's method to determine a root p of an equation $f(x) = 0$. Hence, using initial approximation $p_0 = 2$, perform two iterations to obtain approximation to the value of $\sqrt[3]{17}$.

P.T.O.

2. (*a*) Use Newton's method to approximate the root of the equation $\ln(1 + x) - \cos(x) = 0$, in the interval (0, 1). Use error tolerance = 10^{-4}, and initial approximation $p_0 = 0.5$.

(*b*) Define a fixed point of a function. Let g be a continuous function on the closed interval $[a, b]$, with $g : [a, b] \to [a, b]$. Furthermore, suppose that g' is continuous on the open interval (a, b), and there exists a positive constant k such that $|g'(x)| \leq k < 1, \forall x \in (a, b)$. Prove that if $g'(p) \neq 0$, then for any $p_0 \in [a, b]$, the sequence $\{p_n\}$, where $p_n = g(p_{n-1})$, converges only linearly to the fixed point p.

(*c*) Perform four iterations of Secant method to approximate the root of $x^4 - 7x^2 + 3 = 0$, in the interval (0, 1), using initial approximations $p_0 = 0$ and $p_1 = 1$.

3. (*a*) For the following system of equations :

$$8x + 5y + 2z = 8,$$

$$6x + 9y + 2z = 11,$$

$$4x + 3y + 8z = 1,$$

starting with the initial approximation as (0, 0, 0), using Gauss Jacobi method, find approximate solution by performing three iterations.

(*b*) For the following system of equations :

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18,$$

$$2x - 3y - 20z = 25,$$

starting with initial approximation as (0, 0, 0), using Gauss Seidel method, find the approximate solution by performing three iterations.

(c) Find the LU decomposition by taking $u_{ii} = 1$ for the matrix :

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}.$$

4. (a) Construct the Newton form of interpolating polynomial for a function f passing through the points (0, 1), (1, 3) and (3, 55). Hence, evaluate $f(2.5)$.

(b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use Lagrange interpolation to calculate an approximate value for $f(1.04)$ and obtain a bound on the truncation error at $x = 1.04$.

(c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the data :

x	$f(x)$
1	3
2	7
4	21
8	73

5. (a) Derive the central difference approximation to the second order derivative of a function f at a point $x = x_0$

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}.$$

(*b*) Verify that the following difference approximation

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}$$

for the first order derivative provides the exact value of the derivative, regardless of the values of h, for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

(*c*) By using Trapezoidal rule, approximate the value of the integral $\int_2^3 (x + \ln x)\,dx$. Further, calculate the absolute error, theoretical error bound and compare them.

6. (*a*) Define the degree of precision of a quadrature rule. Further, calculate the degree of precision of Simpson's rule.

(*b*) Use Euler's method to determine the approximate solution of the initial value problem $y'(x) = xy^3 - y$, $0 \le x \le 2$ such that $y(0) = 2$, by taking the step size as $h = 1/2$.

(*c*) Use Modified Euler's method to determine the approximate solution of the initial value problem : $y'(x) = 3x - y/x$, $1 \le x \le 2$, such that $y(1) = 2$, by taking the step size as $h = 1/2$.