Where $F = (x^5 + 10x\,y^2z^2)\,\hat{i} + (y^5 + 10y\,x^2z^2)\,\hat{j} + (z^5 + 10z\,x^2y^2)\,\hat{k}$, and S is the closed hemispherical surface $z = \sqrt{1 - x^2 - y^2}$ together with the disc $x^2 + y^2 \leq 1$ in xy − plane and N is the outward unit normal vector field.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5594 **J**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : **B.A. / B.Sc. (H)**

Semester : IV (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is **NOT** allowed.

1. (a) Let $f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Find the value of $f(0,0)$ for which $f(x,y)$ is continuous at $(0,0)$.

(b) Compute the slope of the tangent line to the graph of $f(x, y) = \dfrac{x^2+y^2}{xy}$ at $P(1, -1, -2)$ in the direction parallel to

(i) XZ plane

(ii) YZ plane.

(c) Find $\dfrac{\partial w}{\partial r}$ where $w = e^{2x-y+3z^2}$ and $x = r + s - t$, $y = 2r - 3s$, $z = \cos(rst)$.

6. (a) Evaluate the surface integral

$$\iint_S g \, dS$$

where $g(x,y,z) = xz + 2x^2 - 3xy$ and S is that portion of the plane $2x - 3y + z = 6$ that lies over the unit square R: $2 \leq x \leq 3$, $2 \leq y \leq 3$.

(b) Evaluate

$$\oint_C \left(\frac{1}{2}y^2 \, dx + z \, dy + x \, dz\right)$$

where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counterclockwise.

(c) Use the divergence theorem to evaluate

$$\iint_S \mathbf{F} \cdot \mathbf{N} \, dS$$

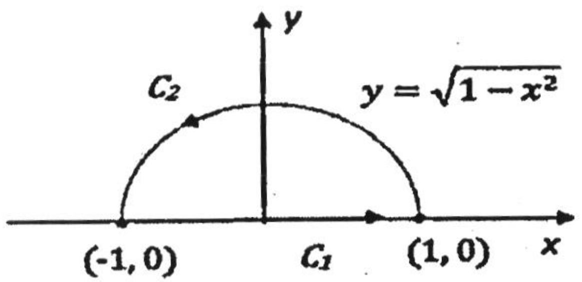
$$F=\left(e^{x}\sin y-y\right)\hat{i}+\left(e^{x}\cos y-x-2\right)\hat{j}$$

is conservative and hence find a scalar potential function f for F.

(c) Verify Green's theorem for the line integral

$$\oint_C(-y\,dx+x\,dy)$$

where C is the closed path shown in the figure below



Path C

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point P(2, 2, −2), find the unit vector pointing in the direction of most rapid increase of f(x, y, z).

(b) Find all the critical points of $f(x,y) = (x-1)(y-1)(x+y-1)$ and classify each as a point of relative maximum, point of relative minimum or a saddle point.

(c) Find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to $4x^2 + 2y^2 + z^2 = 4$.

3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy-plane and above by the graph of $z = 2x + 3y$; R: $0 \le x \le 1$, $0 \le y \le 2$.

(ii) Evaluate $\int_0^1\int_x^1 e^{y^2}\,dydx$.

(b) (i) Sketch the region of integration and write an equivalent integral with the order of integration reversed $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dxdy$.

(ii) Use a double integral for finding the volume of the solid region bounded above by the paraboloid $z = 6 - 2x^2 - 3y^2$ and below by the plane $z = 0$.

(c) Use a double integral to find the area bounded by the curve $r = 1 + \sin\theta$.

4. (a) Use cylindrical co-ordinates to compute the integral

$$\iiint_D z\left(x^2 + y^2\right)^{-1/2} dxdydz$$ where D is the solid bounded above by the plane $z = 2$ and bounded below by the surface $2z = x^2 + y^2$.

(b) (i) Compute the iterated triple integral

$$\int_0^1 \int_0^y \int_0^{\ln y} e^{z+2x}\,dzdxdy.$$

(ii) Evaluate the iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\cos\varphi} \rho^2 \sin\varphi\, d\rho d\theta d\varphi.$$

(c) Evaluate $\iint_R e^{(2y-x)/(y+2x)}\,dA$ where R is the trapezoid with vertices (0,2), (1,0), (4,0) and (0,8).

5. (a) A wire has the shape of the curve

$x = \sqrt{2}\sin t \quad y = \cos t \quad z = \cos t \quad \text{for } 0 \le t \le \pi$

If the wire has density $\delta(x, y, z) = xyz$ at each point (x, y, z), what is its mass?

(b) Show that the vector field