(c) *Find the payoff from a bear spread created using* put options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1358 **J**

Unique Paper Code : 32357614

Name of the Paper : MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (H) Mathematics (CBCS-LOCF)**

Semester : VI – DSE

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator/scientific calculator and normal distribution table is allowed.

1. (a) A bank quotes an interest rate of 14% per annum with quarterly compounding. What is the equivalent rate with

(i) continuous compounding and

(ii) annual compounding?

(b) Explain the following type of interest rates :

(i) Treasury Rates

(ii) LIBOR

(iii) Repo Rates

(c) Portfolio A consists of 1-year zero coupon with a face value of ₹2000 and a 10-year zero coupon bond with face value of ₹6000. Portfolio B consists of a 5.95-year zero coupon bond with face value of ₹5000. The current yield on all bonds is 10% per annum.

(i) Show that both portfolios have the same duration.

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

(b) Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment :

	Fix Rate	Floating Rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

5. (a) *Given that in a risk-neutral world,*

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right],$$ where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(K > S_T)$, the probability that a European put option be exercised in a risk-neutral world.

(b) Calculate the price of a five-month European call option on a non-dividend-paying stock with a strike price of ₹60 when the current stock price is ₹62, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum.

($\ln(62/60) = 0.0328$)

(c) Show that the Black-Scholes-Merton formulas for call and put options satisfy the put- call parity.

(ii) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(You can use the exponential values: e^x = 0.905, 0.368, 0.552, 0.861, 0.223 and 0.409 for x = −0.1, −1.0, −0.595, −0.15, −1.5 and −0.893 respectively)

2. (a) Explain Hedging. How is the risk managed when Hedging is done using?

(i) Forward Contracts

(ii) Options

(b) (i) What is the difference between the over-the counter market and the exchange-traded market?

(ii) An investor enters into a short forward contract to sell 100,000 British pounds for US dollars at an exchange rate of 1.9000 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.8900 and (b) 1.9200?

(c) Write a short note on European call options. Explain the payoffs in different types of call option positions with the help of diagrams.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the vega of a European call and the vega of a European put on a non-dividend-paying stock.

(c) What are the lower and upper bounds for European calls on a non-dividend-paying stock? Calculate the lower bound for the price of a 4-month call option on a non-dividend-paying stock when the stock price is \$28, the strike price is \$25, and the risk-free interest rate is 8% per annum?

4. (a) Consider the standard one-period binomial model where the stock price goes from S_0 to S_0u or S_0d with $d < 1 < u$, and consider an option which pays f_u or f_d in each case, and assume that the interest rate is r and time to maturity is T.

Derive the formula for the no-arbitrage price of the option.

(b) A stock price is currently ₹50. It is known that at the end of six months it will be either ₹45 or ₹55. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹50? (You can use exponential value: $e^{-0.05} = 0.95123$).

(c) A stock price is currently ₹100. Over each of the next two three-month periods it is expected to go up by 8% or down by 7%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month American put option with a strike price of ₹102? (You can use exponential value: $e^{-0.0125} = 0.9876$).