This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7719**

Unique Paper Code : **2352571201**

Name of the Paper : **Elementary Linear Algebra**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor-DSC**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculators is allowed.

1. (*a*) Find a quadratic equation $y = ax^2 + bx + c$ that goes through the points (3, 20), (2, 11) and (–2, 15). 7½

(*b*) State Cauchy-Schwarz inequality and verify it for the vectors $x = (2, -1, 4)$ and $y = (-3, 0, 2)$. 2½+5

(*c*) Use Gaussian elimination method to solve the system of linear equations : 7½

$$x_1 + 2x_2 + 3x_3 = 14$$
$$3x_1 + x_2 + 2x_3 = 11$$
$$2x_1 + 3x_2 + x_3 = 11.$$

P.T.O.

2. (*a*) Find the rank of the matrix : 7½

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}.$$

(*b*) Determine the characteristic roots and characteristic vectors of the matrix : 7½

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

(*c*) Use the Gauss-Jordan method to convert the given matrix to reduced row echelon form : 7½

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

3. (*a*) Use the Diagonalization Method to determine whether the following matrix A is diagonalizable. If so, specify the matrices D and P. Check your work by verifying that $D = P^{-1} AP$: 7½

$$A = \begin{bmatrix} -5 & 18 & 6 \\ -2 & 7 & 2 \\ 1 & -3 & 0 \end{bmatrix}.$$

(*b*) Let V be a vector space. Prove that for any real number a and vector v in V, $av = 0$ implies $a = 0$ or $v = 0$. Show that the set **R** with the usual scalar multiplication but with addition given by $x \oplus y = 2(x + y)$ is not a vector space. 3½+4

(c) Define subspace of a vector space V. Prove that the set of all real valued functions f defined on the interval [0, 1] such that $f\left(\frac{1}{2}\right) = 0$ is a subspace of the vector space of all real valued functions defined on the interval [0, 1]. 2+5½

4. (a) Define span of a subset S in a vector space V. Prove that span (S) forms a subspace of V. 2+5½

(b) Determine whether the given subset of P, the vector space of all polynomials with real coefficients, is linearly independent : 7½

$$\{3x^3 + 2x + 1,\ x^3 + x,\ x - 5,\ x^3 + x - 10\}.$$

(c) Define a basis for a vector space V. Check whether the following subset of $\mathbf{R}^4$ forms a basis of $\mathbf{R}^4$: 2+5½

$$\{[2, 1, 0, 0],\ [0, 1, 1, -1],\ [0, -1, 2, -2],\ [3, 1, 0, -2]\}.$$

5. (a) Define a linear transformation from one vector space to another. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be given by $T(x, y) = (y, x)$ and $S : \mathbf{R}^2 \to \mathbf{R}^2$ given by $S(x, y) = (x + 1, y)$, where $\mathbf{R}^2$ is a vector space over $\mathbf{R}$. Are T and S linear transformations ? Justify your answer. 1½+3+3

(b) Find the matrix of the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined as $T(x, y, z) = (x, y, 0)$ with respect to the standard ordered basis of $\mathbf{R}^3$. Also find the matrix of $[T]_\beta$, where $\beta = \{(0, 0, 1), (1, 0, 0), (0, 1, 0)\}$. Are the two matrices equal ? 3½+3+1

(c) State Dimension Theorem for a linear transformation. Verify Dimension Theorem for the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined as $T(x, y, z) = (0, y)$. 2+5½

6. (a) Is the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$ defined as $T(x, y) = (x, x + y, y)$ one-to-one ? Is it onto ? Justify your answer. 4+3½

(b) Show that $T : M_{2\times2} \to M_{2\times2}$ given by $T(A) = \text{trace } A$ is a linear transformation. Is T an isomorphism ? 3½+4

(c) Find bases for range and kernel of the linear transformation $T : P_4 \to P_2$ given by :

$$T(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4) = a_0 + a_1x + a_2x^2.$$ 4+3½