

This question paper contains 4 printed pages]

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S. No. of Question Paper : 7719

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : B.A./B.Sc. (Prog.) with

Mathematics as Non-Major/Minor-DSC

Semester : II

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts from each question.

All questions carry equal marks.

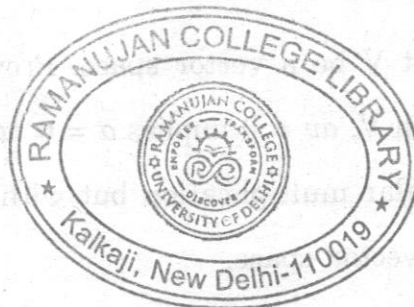
Use of simple calculators is allowed.

1. (a) Find a quadratic equation $y = ax^2 + bx + c$ that goes through the points (3, 20), (2, 11) and (-2, 15). 7½
- (b) State Cauchy-Schwarz inequality and verify it for the vectors $x = (2, -1, 4)$ and $y = (-3, 0, 2)$. 2½+5
- (c) Use Gaussian elimination method to solve the system of linear equations : 7½

$$x_1 + 2x_2 + 3x_3 = 14$$

$$3x_1 + x_2 + 2x_3 = 11$$

$$2x_1 + 3x_2 + x_3 = 11.$$



P.T.O.

2. (a) Find the rank of the matrix :

7½

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}.$$

- (b) Determine the characteristic roots and characteristic vectors of the matrix :

7½

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

- (c) Use the Gauss-Jordan method to convert the given matrix to reduced row echelon form :

7½

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}.$$

3. (a) Use the Diagonalization Method to determine whether the following matrix A is diagonalizable. If so, specify the matrices D and P. Check your work by verifying that $D = P^{-1}AP$:

7½

$$A = \begin{bmatrix} -5 & 18 & 6 \\ -2 & 7 & 2 \\ 1 & -3 & 0 \end{bmatrix}.$$

- (b) Let V be a vector space. Prove that for any real number a and vector v in V, $av = 0$ implies $a = 0$ or $v = 0$. Show that the set \mathbf{R} with the usual scalar multiplication but with addition given by $x \oplus y = 2(x + y)$ is not a vector space.

3½+4

- (c) Define subspace of a vector space V . Prove that the set of all real valued functions f defined on the interval $[0, 1]$ such that $f\left(\frac{1}{2}\right) = 0$ is a subspace of the vector space of all real valued functions defined on the interval $[0, 1]$. 2+5½

4. (a) Define span of a subset S in a vector space V . Prove that $\text{span}(S)$ forms a subspace of V . 2+5½

- (b) Determine whether the given subset of P , the vector space of all polynomials with real coefficients, is linearly independent : 7½

$$\{3x^3 + 2x + 1, x^3 + x, x - 5, x^3 + x - 10\}.$$

- (c) Define a basis for a vector space V . Check whether the following subset of \mathbf{R}^4 forms a basis of \mathbf{R}^4 : 2+5½

$$\{[2, 1, 0, 0], [0, 1, 1, -1], [0, -1, 2, -2], [3, 1, 0, -2]\}.$$

5. (a) Define a linear transformation from one vector space to another. Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be given by $T(x, y) = (y, x)$ and $S : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ given by $S(x, y) = (x + 1, y)$, where \mathbf{R}^2 is a vector space over \mathbf{R} . Are T and S linear transformations ? Justify your answer. 1½+3+3

- (b) Find the matrix of the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ defined as $T(x, y, z) = (x, y, 0)$ with respect to the standard ordered basis of \mathbf{R}^3 . Also find the matrix of $[T]_\beta$, where $\beta = \{(0, 0, 1), (1, 0, 0), (0, 1, 0)\}$. Are the two matrices equal ? 3½+3+1

(c) State Dimension Theorem for a linear transformation. Verify Dimension Theorem for the linear transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined as

$$T(x, y, z) = (0, y). \quad 2+5\frac{1}{2}$$

6. (a) Is the linear transformation $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ defined as $T(x, y) = (x, x + y, y)$ one-to-one ? Is it onto ? Justify your answer. $4+3\frac{1}{2}$

(b) Show that $T : M_{2 \times 2} \rightarrow M_{2 \times 2}$ given by $T(A) = \text{trace } A$ is a linear transformation. Is T an isomorphism ? $3\frac{1}{2}+4$

(c) Find bases for range and kernel of the linear transformation $T : P_4 \rightarrow P_2$ given by :

$$T(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4) = a_0 + a_1x + a_2x^2. \quad 4+3\frac{1}{2}$$

