(b) Find the optimal strategies for both player A and B and value of the game whose payoff matrix is given below.

	B_1	B_2
A_1	1	-3
A_2	3	5
A_3	-1	6
A_4	4	1
A_5	2	2
A_6	-5	0

(c) Solve the following system of equations by using Simplex method :

$$x_1 + x_2 = 1$$

$$2x_1 + x_2 = 3.$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1360 **J**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Define a convex set. Prove that the set S defined as;

$$S = \{(x_1, x_2): x_1^2 + x_2^2 \leq 1\} \text{ is a convex set.}$$

(b) Sketch the feasible region of the given linear programming problem and hence find its optimal solution

$$\text{Max } Z = 3x_1 + x_2$$
$$\text{s.t,} \quad -x_1 + 2x_2 \le 0$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

(c) Reduce $2a_1 + 3a_2 + a_3 = b$ to a Basic Feasible Solution, if

$a_1 = [2,3]$, $a_2 = [1,1]$, $a_3 = [4,5]$ and $b = [11,14]$.

Name the basic and non-basic variables in the reduced Basic Feasible Solution.

2. (a) Apply Two-phase method to find the solution of the following linear programming Problem :

$$\text{Min } Z = 5x_1 + 11x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \le 4$$
$$3x_1 + 4x_2 \ge 24$$
$$2x_1 - 3x_2 \ge 6$$
$$x_1, x_2 \ge 0.$$

	Depots				
Factories	D_1	D_2	D_3	D_4	Capacity (tons)
R_1	5	7	13	10	700
R_2	8	6	14	13	400
R_3	12	10	9	11	800
Requirements	200	600	700	400	

Determine the optimum distribution for this company to minimize the shipping cost using North-West Comer Rule, Least Cost Method and Vogel's Approximation Method. Compare the solutions (in terms of costs).

5. (a) Define saddle point. Solve the game whose payoff matrix is given below.

	Player B			
Player A	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

(b) A company wish to hire five employees for five positions claim that they have the required knowledge for each position. Each position can be assigned to one and only one employee. The cost of assignment (per lakh CTC) of position to each employee is given below in the table. Allocate the positions to appropriate employee for optimal assignment.

Employee \ Positions	A	B	C	D	E
I	3	8	2	10	3
II	8	7	2	9	7
III	6	4	2	7	5
IV	8	4	2	3	5
V	9	10	6	9	10

(c) An Oil company has three refineries and four depots. The transportation cost per ton, capacity and requirements are given below in the table.

(b) Solve the following linear programming problem by Simplex method :

$$\begin{aligned} \text{Max } Z &= 3x_1 + 2x_2 \\ \text{s.t, } \quad x_1 &\le 4 \\ x_2 &\le 6 \\ x_1 + x_2 &\le 5 \\ x_1, x_2 &\ge 0. \end{aligned}$$

(c) Using Chame's M-technique to solve the following linear programming problem:

$$\begin{aligned} \text{Max } Z &= x_1 + 2x_2 + 3x_3 \\ \text{s.t, } \quad x_1 + x_2 + x_3 &= 5 \\ 2x_2 + x_3 &\ge 2 \\ 2x_1 + x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

3. (a) Find the dual of the following linear programming problem:

$$\begin{aligned} \text{Max } Z &= 5x_1 + 6x_2 \\ \text{s.t, } \quad x_1 + 2x_2 &= 5 \\ -x_1 + 5x_2 &\ge 3 \\ 4x_1 + 7x_2 &\le 8 \\ x_1 \text{ unrestricted}, \ x_2 &\ge 0. \end{aligned}$$

(b) Solve graphically the dual of given primal linear programming problem, then use complementary slackness condition to determine the solution of the primal problem.

$$\text{Min } Z = 2x_1 + x_2 + 3x_3 + 6x_4$$
$$\text{s.t.} \quad x_1 + x_2 + 3x_3 + 2x_4 \geq 3$$
$$2x_1 + x_2 + x_3 + 3x_4 \geq 2$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

(c) Let x_0 be the feasible solution to the primal problem.

$$\text{Max } Z = CX$$
$$\text{s.t.} \quad AX \leq b$$
$$X \geq 0.$$

Where, $X^T, C \in \mathbb{R}^n$, $b^T \in \mathbb{R}^m$ and A = m × n matrix. If w_0 be a feasible solution to the dual of the primal problem.

$$\text{Min } Z^* = b^T w$$
$$\text{s.t.} \quad A^T w \geq C^T$$
$$w \geq 0.$$

Where $w^T \in \mathbb{R}^m$. Then prove that $Cx_0 \leq b^T w_0$.

4. (a) A company has three production houses and supplying the product to five warehouses. The cost of production varies from production houses to production houses and cost of transportation from production houses to warehouses also varies. Each Production houses has a certain production capacity and each warehouses have certain amount of requirement. The cost of transportation per unit is given below.

	Warehouse					
Production House	I	II	III	IV	V	Supply
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	

Determine the optimum quantity to be supplied from each production houses to different warehouses at minimum cost.