| Player A | Player B $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 3 | 2 | 4 |
| $A_2$ | -1 | 4 | 2 |
| $A_3$ | 2 | 2 | 6 |

(b) For the following Payoff table, transform the zero-sum game into an equivalent linear programming problem.

| Player A | Player B $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 9 | 1 | 4 |
| $A_2$ | 0 | 6 | 3 |
| $A_3$ | 5 | 2 | 8 |

(c) Find the range of p and q that will render the entry (2, 2) a saddle point for the game.

| Player A | Player B I | II | III |
|---|---|---|---|
| I | 2 | 4 | 5 |
| II | 10 | 7 | q |
| III | 4 | p | 6 |





[This question paper contains 8 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 4766** **J**

Unique Paper Code : 2354002004

Name of the Paper : Linear Programming

Name of the Course : **Mathematics: Generic Elective**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting two parts from each question.

3. All questions carry **equal** marks.

4. Use of calculator is not allowed.

1. (a) Solve the following linear programming problem graphically:

$$\text{Minimize } z = x_1 + 2x_2$$

*Subject to*

$$x_1 + x_2 \geq 1$$
$$100x_1 + 10x_2 \geq 50$$
$$10x_1 + 100x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

(b) Find all basic feasible solutions of the following system of equations:

$$3x_1 + 2x_2 - 7x_3 = 1$$
$$x_1 - 5x_2 - 6x_3 = -4$$

(c) Show that the set $S = \{(x, y) \in R^2 \mid 2x^2 + 3y^2 \leq 6\}$ is a convex set.

2. (a) Solve the following linear programming problem:

$$\text{Maximize } z = 3x_1 + 2x_2 + x_3$$

*Subject to*

$$2x_1 - 3x_2 + 2x_3 \leq 3$$
$$x_1 + x_2 + x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$



**Jobs**

| Machines | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
|---|---|---|---|---|---|
| A | 2.5 | 5 | 1 | 6 | 1 |
| B | 2 | 5 | 1.5 | 7 | 3 |
| C | 3 | 6.5 | 2 | 8 | 3 |
| D | 3.5 | 7 | 2 | 9 | 4.5 |
| E | 4 | 7 | 3 | 9 | 6 |

(b) Explain the Two Person Zero Sum games and saddle point in game theory. Further differentiate between pure strategy and mixed strategy.

(c) Consider the game with the following payoff table:

| Player A \ Player B | $B_1$ | $B_2$ |
|---|---|---|
| $A_1$ | 2 | 6 |
| $A_2$ | -2 | $\lambda$ |

(i) Show that the game is strictly determinable, whatever $\lambda$ may be.

(ii) Determine the value of game.

6. (a) Use dominance rule to reduce the size of the following payoff matrix to (2 x 2) size and hence, find the optimal strategies and value of the game.

| | P | Q | R | Supply |
|---|---|---|---|---|
| **A** | 6 | 12 | 8 | 500 |
| **B** | 8 | 14 | 12 | 300 |
| **C** | 12 | 12 | 14 | 400 |
| **Demand** | 350 | 600 | 250 | |

(c) Solve the following cost minimization Assignment Problem of assigning Jobs to Machines:

**Jobs**

| | | $J_1$ | $J_2$ | $J_3$ |
|---|---|---|---|---|
| **Machines** | $M_1$ | 18 | 8 | 10 |
| | $M_2$ | 24 | 13 | 15 |
| | $M_3$ | 28 | 17 | 19 |
| | $M_4$ | 32 | 19 | 22 |

5, (a) A Company is faced with the problem of assigning five different machines to five different jobs. The costs are estimated as follows (in hundreds of rupees). Solve the problem assuming that the objective is to minimize the total cost:



(b) Solve the following linear programming problem:

$$\textit{Maximize } z = x_1 + 12x_2 + 9x_3$$

*Subject to*

$$3x_1 + 2x_2 - 6x_3 \leq 20$$
$$2x_1 + 6x_2 + 3x_3 \leq 30$$
$$6x_1 \quad + 2x_3 \leq 16$$
$$x_1, x_2, x_3 \geq 0$$

(c) Using Simplex method, solve the following linear programming problem and show that it has an unbounded solution:

$$\textit{Minimize } z = -5x_1 + 4x_2 + x_3$$

*Subject to*

$$x_1 + x_2 - 3x_3 \leq 8$$
$$2x_2 - 2x_3 \leq 7$$
$$-x_1 - 2x_2 + 4x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

3. (a) Using Simplex method, solve the following linear programming problem and show that it has no solution:

*Minimize* $z = x_1 + x_2 + x_3$

*Subject to*

$$-x_1 + 2x_2 + x_3 \leq 1$$
$$-x_1 + 2x_3 \geq 4$$
$$x_1 - x_2 + 2x_3 = 4$$
$$x_1, x_2, x_3 \geq 0$$

(b) Write the dual of the following linear programming problem:

*Minimize* $z = 2y_1 - 3y_2 + 4y_3$

*Subject to*

$$8y_1 - y_3 = 50$$
$$6y_2 + y_3 \leq 60$$
$$2y_1 - 3y_2 + 6y_3 \geq 30$$
$$y_1, y_2 \geq 0, -23 \leq y_3 \leq 0$$

(c) Show that the dual of the dual of the following MAX problem is the MAX problem itself:



*Maximize* $z = x_1 - 2x_2 - 3x_3$

*Subject to*

$$x_2 + 2x_3 \geq 1$$
$$x_1 + 3x_3 \leq 2$$
$$2x_1 - 3x_2 = 3$$
$$x_1, x_2 \geq 0, x_3 \text{ unrestricted}$$

4. (a) Obtain an initial basic feasible solution for the following cost minimization transportation problem using

(i) North West Corner Rule

(ii) Least Cost Method

| | **P** | **Q** | **R** | **S** | **Supply** |
|---|---|---|---|---|---|
| **A** | 4 | 6 | 8 | 13 | 50 |
| **B** | 13 | 11 | 12 | 10 | 70 |
| **C** | 14 | 3 | 15 | 13 | 30 |
| **D** | 9 | 17 | 13 | 18 | 50 |
| **Demand** | 40 | 35 | 105 | 20 | |

(b) Obtain an initial basic feasible solution for the following cost minimization transportation problem using Vogel's Approximation Method and then find the optimal solution by MODI method.