

(c) Prove that in a ring R with unity 1 ,

- (i) if 1 has infinite order under addition, then the characteristic of R is 0 ,
- (ii) if 1 has finite order n under addition, then the characteristic of R is n .

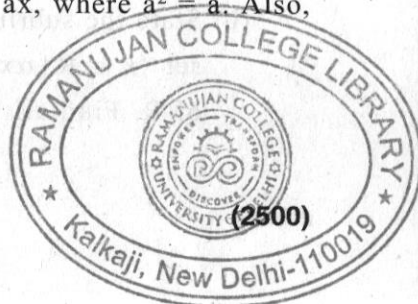
Hence or otherwise state the characteristics of the rings \mathbb{Z} , $M_2(\mathbb{Z})$ and $\mathbb{Z}_3[i]$.

6. (a) State the ideal test. Let R be a commutative ring with unity and let $a \in R$. Then verify that the sets $I = \{ra \mid r \in R\}$ and $S = \{ar \mid r \in R\}$ are ideals of R .

(b) Let $\phi: R \rightarrow S$ be a ring homomorphism. Prove that :

- (i) $\phi(nr) = n\phi(r)$ and $\phi(r^n) = (\phi(r))^n$ for any $r \in R$ and any positive integer n .
- (ii) $\phi(R)$ is commutative if R is commutative.

(c) Prove that every ring homomorphism ϕ from \mathbb{Z}_n to itself has the form $\phi(x) = ax$, where $a^2 = a$. Also, find $\text{Ker}\phi$.



[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5106

J

Unique Paper Code : 2354000009

Name of the Paper : Abstract Algebra

Name of the Course : COMMON PROG GROUP

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Each part carries **7.5** marks.
5. Use of Calculator not allowed.

1. (a) Define $SL(2, \mathbb{F})$ and find the inverse of the

element $\begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$ in $SL(2, \mathbb{Z}_5)$.

- (b) Describe the symmetries of a square. Construct the corresponding Cayley Table.

- (c) Prove that the set $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$ is a group under matrix multiplication.

2. (a) If H and K are subgroups of a group G , show that $H \cap K$ is a subgroup of G . What can you say about $H \cup K$? Justify.

- (b) Let G be a group. Then prove that: $c(a) = c(a^{-1})$ $\forall a \in G$, where $c(a)$ is the centralizer of a in G .

- (c) Prove that the center of a group G is a subgroup of G . Find the center of the group D_4 . Justify.

3. (a) Define a cyclic group. Find an example of a noncyclic group, all of whose proper subgroups are cyclic. Justify your answer.

- (b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{pmatrix}$,

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}.$$

Write α , β and $\alpha\beta$ as product of disjoint cycle and also find β^{-1} .

- (c) State the Lagrange's Theorem. Let $|G| = 60$, then what are the possible orders for subgroups of G .

4. (a) Let H be a subgroup of a group G , then show that either $\alpha H = bH$ or $\alpha H \cap bH = \phi$ for every $\alpha, b \in G$. Also, find all the left cosets of $\{1, 11\}$ in $U(30)$.

- (b) Define the factor group. Also, let $G = \mathbb{Z}_{24}$, then find the order of the element $14 + \langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$.

- (c) Consider the map $f: \mathbb{Z} \rightarrow \mathbb{Z}_n$ defined by $f(m) = m \bmod n$. Show that f is a homomorphism. Also, find the $\text{Ker } f$.

5. (a) Show that the set $\mathbb{Z}[x]$ of all polynomials in the variable x with integer coefficients under addition and multiplication is a commutative ring with unity $f(x) = 1$.

- (b) State the subring test. Prove that in a ring R , the set $\{x \in R \mid \alpha x = x\alpha \text{ for all } \alpha \in R\}$ is a subring of R . Find all the subrings of the ring \mathbb{Z} .