This question paper contains 3 printed pages]

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S. No. of Question Paper : **7439**

Unique Paper Code : **2352572401**

Name of the Paper : **Abstract Algebra**

Name of the Course : **B.A./B.Sc. (Programme) Mathematics as Non-Major/Minor-DSC**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

Attempt *all* questions by selecting any *two* parts from each question.

Each part carries **7.5** marks.

Use of calculator is not allowed.

1. (*a*) Let $a$ and $n$ be positive integers and let $d = gcd(a, n)$. Prove that the equation $ax \bmod n = 1$ has a solution if and only if $d = 1$.

   (*b*) Define Group. Show that the general linear group of $2 \times 2$ matrices over **R** is a group.

   (*c*) Prove that a group G of order 3 must be cyclic.



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2. (*a*) Define Abelian group. Show that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a$, $b$ in G.

(*b*) Define order of an element in a group. Find the order of each element of U(15).

(*c*) State and prove one-step subgroup test. Use it to prove that if G is an abelian group with identity $e$, then $H = \{x \in G \mid x^2 = e\}$ is a subgroup of G.

3. (*a*) If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{pmatrix}$, then :

(*i*) Find $O(\alpha\beta)$

(*ii*) Find $O(\beta\alpha)$

(*iii*) Find $\alpha^{-1}$.

(*b*) Find all the right cosets of {1, 15} in U (32).

(*c*) Show that the factor group $\mathbf{Z}/5\mathbf{Z}$ is a cyclic group of order 5.

4. (*a*) Define normal subgroup. Prove that SL(2, **R**), the group of 2 × 2 matrices with determinant 1 is normal subgroup of GL(2, **R**), the group of 2 × 2 matrices with non-zero determinant.

(*b*) Show that $\phi : Z_{12} \to Z_{12}$ given by $\phi(x) = 4x$ is a group homomorphism.

(*c*) Show that the kernel of homomorphism $\phi : \mathbf{R}^* \to \mathbf{R}^*$ given by $\phi(x) = |x|$ is a cyclic group.

5. (*a*) Prove that $Z_4$ is a ring and find its set of units.

(*b*) Let $x$, $y$ and $z$ belong to a ring R. Then show that :

(*i*) $(-x)(-y) = xy$

(*ii*) $x(y - z) = xy - xz$.

(*c*) State ideal test and prove that $2\mathbf{Z} = \{2n \mid n \in \mathbf{Z}\}$ is an ideal of $\mathbf{Z}$.

6. (*a*) Define zero-divisors and integral domain. Give an example of a commutative ring without zero-divisors that is not an integral domain.

(*b*) Let $a$, $b$ and $c$ belong to an integral domain. If $a \neq 0$ and $ab = ac$, then show that $b = c$.

(*c*) Let $f$ be a ring homomorphism from a ring R to a ring S and let A be a subring of R. Then prove that $f(\mathrm{A}) = \{f(a) \mid a \in \mathrm{A}\}$ is a subring of S.