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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1401**

Unique Paper Code : **2352011103**

Name of the Paper : **DSC 3-Probability and Statistics**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (*a*) Construct a stem-and-leaf display for the given batch of exam scores, repeating each stem twice. What feature of the data is highlighted by this display ?

74	**89**	**80**	**93**	**64**	**67**	**72**	**70**	**66**	**85**	**89**	**81**
81	**71**	**74**	**182**	**85**	**63**	**72**	**81**	**81**	**95**	**84**	
81	**80**	**70**	**69**	**66**	**60**	**83**	**85**	**98**	**84**	**68**	**90**
82	**69**	**72**	**87**	**88**							

(*b*) The following data gives the reasonable award (in $ 1000s) in 27 cases identified by the court :

37	60	75	115	135	140	149	150	238	290	340	410
600	750	750	750	1050	1100	1139	1150	1200	1200	1250	
1576	1700	1825	2000								

(*i*) What are the values of the quartiles and what is the value of the fourth spread ?

(*ii*) How large or small does an observation have to be to qualify as an outlier ? As an extreme outlier ?

(*c*) The value of Young's modulus (GPa) was determined for cast plates consisting of certain intermetallic substrates, resulting in the following sample observations :

116.6 115.9 114.6 115.2 115.8.

(*i*) Calculate the sample mean, the sample variance and the sample standard deviation.

(*ii*) Subtract 100 from each observation to obtain a sample of transformed values. Now calculate the sample variance of these transformed values and compare it with sample variance for the original data.

2. (*a*) A certain system can experience three different types of defects. Let A_i (i = 1,2,3) denote the event that the system has a defect of type i. Suppose that $P(A_1) = .12$, $P(A_2) = .07$, $P(A_3) = .05$, $P(A_1 \cup A_2) = .13$, $P(A_1 \cup A_3) = .14$, $P(A_2 \cup A_3) = .10$, $P(A_1 \cap A_2 \cap A_3) = .01$

(*i*) What is the probability that the system has both type-1 and type-2 defects but not a type-3 defect ?

(*ii*) What is the probability that the system has at most of these defects ?

(*b*) State Bayes' Theorem. At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2) and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested ?

(*c*) If A and B are independent event, show that :

(*i*) A' and B are also independent

(*ii*) A and B' are also independent

(*iii*) A' and B' are also independent.

3. (*a*) Starting at a fixed time, observe the gender of each new-born child at a certain hospital until a boy is born. Assume that the successive births are independent and define the random variable X by X = number of births observed.

Find :

(*i*) The probability mass function (*pmf*) of X

(*ii*) The cumulative distribution function (*cdf*) of X.

(*b*) A certain brand of upright freezer is available in three different rated capacities : 450L, 500L and 550L. Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has *pmf* $p(450) = .2$, $p(500) = .5$, $p(550) = .3$.

(*i*) Compute E(X), $E(X^2)$ and V(X).

(*ii*) If the price of a freezer having capacity X is 2.5X – 650, what is the expected price paid by the next customer to buy a freezer ?

(*c*) Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter $\mu = 20$. What is the probability that the number of drivers travelling between the origin and the destination during the designated time will be :

(*i*) between 10 and 20, both inclusive ?

(*ii*) within 2 standard deviations of the mean value ?

4. (*a*) "Time headway" in traffic flow is the elasped time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars. Suppose that in a different traffic environment, the distribution of time headway has the form

$$f(x) = \begin{cases} \dfrac{K}{x^4}, & x > 1 \\ 0, & x \le 1. \end{cases}$$

(*i*) Determine the value of K for which $f(x)$ is a *pdf*.

(*ii*) Obtain the *cdf*.

(*iii*) Determine $P(X > 2)$ and $P(2 < X < 3)$.

(*b*) In a road-paving process, asphalt mix is delivered to the hopper of the paver by truck that haul the material from the batching plant. Let the random variable X = truck haul time can be modelled with a normal distribution with mean value 8.46 min and standard deviation .913 min time. What is the probability that haul time :

(*i*) is at least 10 min ?

(*ii*) exceeds 15 min ?

(*iii*) remains between 8 and 10 min ?

(*c*) Suppose component lifetime is exponentially distributed with parameter λ. After putting the component into service, leave for a period of t_0 hours and then return to find the component still working. What is the probability that it lasts at least an additional t hours ?

5. (*a*) Suppose that 25 percent of all students at a large public university receive financial aid. For a random sample of students of size 50, use normal approximation of binomial distribution to find the probability that :

(*i*) at most 10 students receive aid,

(*ii*) between 5 and 15 (both inclusive) of the selected students receive aid.

(*b*) The stress range in certain railway bridge connection is exponentially distributed with an average of 6 MPa (megapascals). Find the probability that :

(*i*) The stress range is at most 10 MPas,

(*ii*) The stress range is between 5 and 10 MPas (both inclusive).

(*c*) Let X be the temperature in degree Celsius at which a certain chemical reaction takes place and let Y be the same temperature in degree Fahrenheit. It is known that conversion of unit from one of the two units to the other unit follows the rule Y = 1.8 X + 32.

(*i*) If the median of the Y distribution is 50, find the median of X distribution.

(*ii*) If third quartile for X distribution is 15, find the third quartile for Y distribution.

6. (*a*) The lifetime of a certain type of battery is normally distributed with mean value 10 hours and standard deviation 1 hour. There are four batteries in a package. What lifetime value is such that the total lifetime of all batteries in a package exceeds that value for only 5 percent of all packages ?

(*b*) The efficiency ratio for a steel specimen immersed in a phosphating tank is the weight of the phosphate coating divided by the metal loss (both in mg/ft 2). The following data is on tank temperature (x) and efficiency ratio (y) :

Temp.(x)	170	172	173	174	174	175	176
Ratio(y)	.84	1.31	1.42	1.03	1.07	1.08	1.04
Temp.(x)	177	180	180	180	180	180	181
Ratio(y)	1.80	1.45	1.60	1.61	2.13	2.15	.84
Temp.(x)	181	182	182	182	182	184	184
Ratio(y)	1.43	.90	1.81	1.94	2.68	1.49	2.52
Temp.(x)	185	186	188				
Ratio(y)	3.00	1.87	3.08				

(*i*) Determine the equation of the estimated regression line.

(*ii*) Calculate a point estimate for true average efficiency ratio when tank temperature is 182.

(*c*) If x and y are two variables such that $\sum x = 25.7$, $\sum x^2 = 88.31$, $\sum y = 14.40$, $\sum y^2 = 26.4324$, $\sum xy = 46.856$, where summation runs over $n = 15$ values of x and y, then compute the coefficient of correlation between x and y. Does the value of the coefficient of correlation between the variables change when each value of x and y in the data is doubled ?