(b) Prove that a graph G is bipartite if and only if it does not contain an odd cycle. (8)

[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5527 **J**

Unique Paper Code : 2342012401

Name of the Paper : Design and Analysis of Algorithms

Name of the Course : **B.Sc. (H) Computer Science**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is Compulsory.
3. Attempt any **four** questions from **Section B**.
4. Parts of a question must be answered together.

Section A

1. (a) Consider the recurrence relation for Strassen's matrix multiplication algorithm. (2)

$$T(n) = 7T(n/2) + \Theta(n^2)$$

Explain which operations contribute to the cost component $\Theta(n^2)$.

(b) Given that Vertex Cover $\leq$ p Independent Set, fill the blanks in the following statement with these terms appropriately : Vertex Cover, Independent Set. (2)

Statement: _____ is at least as hard as _____ .

(c) Demonstrate through an array of length 5 that the selection sort algorithm is not stable. (3)

(d) Give an example of a 4-node graph that does not have a topological ordering. (3)

he should pursue for the specified scenario.

S. NO.	COURSE	FEE	CREDIT
1	OC1	40	12
2	OC2	20	8
3	OC3	30	16
4	OC4	10	9
5	OC5	30	5

(7)

(b) Let G = (V, E) be an undirected graph. Prove that a subset $S \subseteq V$ is an independent set if and only if its complement V\S is a vertex cover and vice-versa. (8)

7. (a) A group of people wants to cross a river. There are several boats available each with a different upper limit on the number of people it could accommodate. Each boat costs the same and you wish to hire minimum number of boats. Give a greedy strategy that works and discuss its time complexity. (7)

Adv No.	Start Time	Finish Time	Payment
A1	0	3	3
A2	1	4	2
A3	0	5	4
A4	3	6	1
A5	4	7	2
A6	3	9	5
A7	5	10	2
A8	8	10	1

6. (a) A student named Bhanu aims to improve his skills by enrolling in several online courses. Each course requires a registration fee, and upon successful completion, awards a certain number of credits. Bhanu has a limited amount of savings and must select from the available courses in a way that maximizes the total credits earned while staying within his budget.

Given that Bhanu has a budget of Rs. 70, employ an appropriate algorithm to identify the courses

(e) Modify the counting sort algorithm to sort an array that includes negative integers in the range [-50, 50]. (3)

(f) Consider an optimal solution O to an unweighted interval scheduling problem on set of intervals S. Let i be the interval in O that finishes first. Further, let $S' \subseteq S$ be the subset of intervals compatible with i. Show that $O \setminus \{i\}$ must be an optimal solution to interval scheduling problem on S'. (4)

(g) Explain the terms : (4)

(i) NP Hard problem

(ii) NP Complete problem

(h) Show that the greedy method based on profit-to-weight ratio does not always give an optimal solution for the 0-1 knapsack problem? (4)

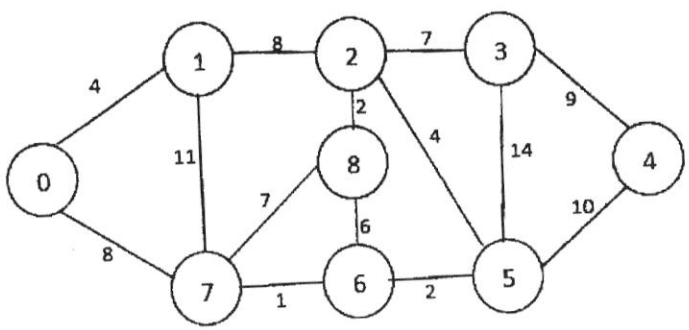
(i) Establish the correctness of the insertion sort algorithm using an appropriate loop invariant. (5)

Section B

2. (a) Suppose an n × n array A consists of 1's and 0's such that in any row of A all the 1's come before any 0's in that row. Give an $O(n \log_2 n)$ algorithm for counting the number of 1's in A. (5)

(b) Given an undirected graph G = (V, E), a source node S and a target node T, can we use the DFS algorithm to find the shortest path from S to T? Justify your answer. (5)

(c) For the graph given below, find a minimum spanning tree using the Prim's algorithm considering node 0 as the start node. Also, state the time complexity of the same. (5)



5. (a) Foreach of the following scenarios, construct an input example of size 8 that demonstrates the specified behaviour of the merge procedure:

(i) Best-case behaviour

(ii) Worst-case behaviour

Also, compute the total number of comparisons performed for each of the above cases. (7)

(b) You are given a list of advertisements, each with a start time, finish time, and the payment offered. No two ads can overlap in time. Your task is to select a subset of non-overlapping advertisements that maximizes the total payment.

(i) Find the maximum total payment.

(ii) Determine a subset of advertisements that maximizes the total payment. (8)

```
SELECT_NEW(A, p, r, i)
  If p == r
     return A[p]
  x = MEDIAN(A, p, r)
  q = PARTITION-AROUND(A, p, r, x)
  k = q - p + 1
  if i == k
     return A[q]
  elseif i < k
     return SELECT_NEW(A, p, q-1, i)
  else
     return SELECT_NEW(A, q+1, r, i-k)
```

Give the recurrence for the worst-case running time of the above algorithm and solve it.

3. (a) Compare the worst-case time complexities of the standard and randomized versions of Quick Sort algorithm. In standard version of Quicksort, assume that the last element is always chosen as the pivot. Explain how the use of randomization impacts the worst-case time complexity. (5)

(b) You are helping a child plan his cartoon-watching schedule. Each cartoon show has a start time and end time, and the child can watch only one show at a time. The goal is to watch maximum number of cartoons shows. The child suggests watching the longest cartoon first. You suspect this may not lead to watching the maximum number of shows. (5)

(i) Give an example wherein choosing the longest cartoon first does not result in maximum shows being watched.

(ii) Suggest a better strategy to help the child watch as many different shows as possible.

(c) Run the build-max-heap procedure (that internally uses the max-heapify procedure) on the following array and report the number of swap operations done: < 89, 19, 50, 27, 32, 65, 2, 5, 7> (5)

4. (a) Consider a stack that supports the following three operations : (7)

- PUSH(x): Pushes an element x onto the stack.
- POP(): Removes the top element from the stack.
- COPY(): Copies the current contents of the stack into an array and empties the stack. The cost of COPY is proportional to the current size of the stack.

Now, assume that after every sequence of exactly k push and pop operations (in any order), a copy operation is invoked. Further, COPY() is not called at any other time. Thus, the number of elements in the stack never exceeds k.

You are given a sequence of n push, pop and copy operations on an initially empty stack (where n is large and includes many copy operations). Use the accounting method of amortized analysis to show that the total cost of performing this sequence of n operations is O(n), even though the copy operation can have a worst-case cost of O(k).

(b) Suppose you have a function MEDIAN that returns the index of the median of a given array in linear time in the worst case.

(i) Use it to modify the QUICKSORT algorithm to make it run in $O(n \log_2 n)$ time in the worst case.

(ii) Consider the following algorithm that uses the function MEDIAN as a black-box to solve the selection problem for finding the i^{th} order statistic in array A[p: r].