[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 6003 **J**

Unique Paper Code : 2922062401

Name of the Paper : Quantitative Techniques for Management

Name of the Course : **BMS**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) Attempt any **five** questions in all.

(iii) All questions carry equal marks.

(iv) Use of Simple calculators are allowed.

(v) Use of standard normal table is allowed.

1. Solve the following linear programming problem using Big M Method:

Maximize $Z = 2x_1 + 4x_2$

Subject to constraints

$2x_1 + x_2 \leq 18$

$3x_1 + 2x_2 \geq 30$

$x_1 + 2x_2 = 26$

$x_1 , x_2 \geq 0$

(a) Determine the optimal solution and the corresponding value of the objective function.

(b) Assess whether the obtained solution is unique. Provide a justification for your answer.

(c) Interpret the values of the slack, surplus, and artificial variables at the optimal solution. What do they signify in the context of the constraints? (18)

2. A bakery produces two types of cakes: Chocolate Cake and Vanilla Cake, with production costs of Rs. 40 and Rs. 90 per unit, respectively. The bakery faces the following production constraints: the difference between the number of Chocolate Cake and twice the number of Vanilla Cakes should not exceed 8 units; a combination of 3 times the number of Chocolate Cake and 4 times the number of Vanilla Cakes must be exactly equal to 180 units; and a combination of 6 times the number of Chocolate Cakes and 9 times the number of Vanilla Cakes should not exceed 450 units. Additionally, the production quantities cannot be negative. The bakery aims to determine the optimal production quantities of Chocolate and Vanilla Cakes while adhering to these constraints in order to minimize the total cost.

(a) Formulate the above problem as a Linear Programme Problem.

(b) Write the dual of the Linear Programme Problem.

(c) Explain the relationship between a primal and its dual. (18)

3. (a) Two fast food chains, X and Y, are competing to attract more customers in a metropolitan area. Company X uses two promotional strategies, Discount Coupons (X1) and Loyalty Programs (X2). Company Y uses the strategies Free Delivery (Y1), Combo Offers (Y2), Late-night Services (Y3), and Mobile App Promotions (Y4). The gain or loss in customer base (in percentage points) for Company Y is shown in the following payoff matrix. A positive value indicates a gain for Company Y, while a negative value indicates a loss for Company Y.

	X1 (Discount Coupons)	X2 (Loyalty Programs)
Y1 (Free Delivery)	2	-1
Y2 (Combo Offers)	-2	3
Y3 (Late-night Services)	1	0
Y4 (Mobile App Promo)	0	2

Solve the game graphically from the perspective of Food chain X and determine the value of the game. Find the optimal mixed strategy for both chains X and Y. (9)

(b) Consider the following linear programming problem:

Maximize $Z = 40x_1 + 60x_2 + 50x_3$

Subject to:

$x_1 + 2x_2 + x_3 \leq 100$ (Constraint 1 – Machine A)

$3x_1 + 2x_2 + 2x_3 \leq 180$ (Constraint 1 – Machine B)

$x_1, x_2, x_3 \geq 0$

The Optimal solution to the above is $x_1 = 0$, $x_2 = 10$, and $x_3 = 80$. The optimal value of Z = Rs. 4600

Shadow Prices and RHS Sensitivity:

Constraint	RHS	Shadow Price	Allowable Increase	Allowable Decrease
Machine A	100	Rs. 10	80 units	10 units
Machine B	180	Rs. 5	20 units	80 units

Based on the above, answer the following questions:

(i) Interpret the shadow prices of Machine A and Machine B in economic terms.

(ii) What is the significance of the allowable decrease in the right-hand side (RHS) of a constraint in post-optimality analysis? If the availability of Machine A decreases from 100 hours to 80 hours, what will be impact on the optimal value of the objective function (Z)?

(iii) What is the significance of the allowable increase in the RHS a constraint in post-optimality analysis? If the availability of Machine B increases from 180 hours to 200 hours, what will be the impact on the optimal value of Z? (3×3=9)

4. (a) Explain the Maximin and Minimax principles in the context of two-person zero-sum games. How do these principles help in determining the optimal strategies for the players? Support your explanation with a suitable numerical example. (9)

(b) A company has three factories (F1, F2, F3) and four warehouses (W1, W2, W3, W4). The supply capacities of the factories and the demands at the warehouses are given below, along with the unit transportation costs. However, certain routes are prohibited due to logistical constraints. The entry '–' in the cost matrix represents a prohibited route.

	W1	W2	W3	W4	Supply
F1	6	3	–	5	30
F2	5	7	6	2	50
F3	4	–	3	4	40
Demand	20	10	25	35	

First find an initial solution to the problem using Least Cost Method (LCM) or Vogel's Approximation Method (VAM) and them determine optimal solution using the Modified Distribution Method (MODI). (9)

5. (a) A company has identified the following details of tasks which must be completed for the successful completion of a project:

Activity	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Precedence	-	-	A	B	C	C	D, E	F, G
Duration (in days)	4	7	3	2	5	4	9	6

Compute the earliest start (ES), earliest finish (EF), latest start (LS), latest finish (LF) time, and total slack (or float) for each activity in the project. Based on this on this information, determine the critical path and the total duration of the project. (9)

(b) A project has the following details:

Activity	**1-2**	**1-3**	**2-4**	**3-4**	**3-5**	**4-6**	**5-6**
Optimistic Time (in days)	2	5	3	1	4	3	4
Most likely Time (in days)	4	8	4	5	6	6	6
Pessimistic Time (in days)	6	11	11	12	8	9	14

Find the critical path, expected project completion time and the expected variance in completing the project. What is the likelihood of completing the project between 19 and 23 days? If the manager wants to be 95 per cent sure of timely completion, how many days prior to the deadline should he start the project? (9)

6. (a) A project has the following details:

Activity	**1-2**	**2-3**	**2-4**	**3-4**	**4-5**	**4-6**	**5-7**	**6-7**
Normal Time (in days)	14	17	13	12	15	14	19	16
Crash Time (in days)	13	15	12	12	14	12	16	14
Normal cost (in Rs.)	2,700	3,000	2,500	2,250	1,800	2,200	5,100	4,000
Crash cost (in Rs.)	3,150	3,750	3,100	2,250	2,600	2,750	6,750	4,800

Find the critical path, project completion time and the normal cost of completing the project. If the project is to be crashed by 2 days and the overhead cost per week is Rs. 1,000, compute the associated cost.

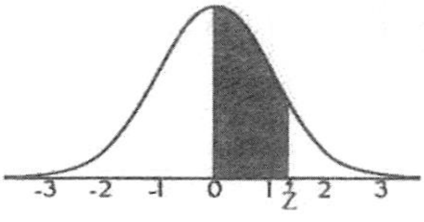
(9)

(b) Given below is the detail of worker-job combinations and the associated satisfaction scores (out of 15):

Worker/Job	I	II	III	IV	V
A	5	11	10	12	4
B	2	4	6	3	5
C	3	12	5	14	6
D	6	14	4	11	7
E	7	9	8	12	5

Find the optimal worker-job assignment that maximizes the satisfaction score of workers.

(9)



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998