future observation y_0 corresponding to a specified level x_0 of the regressor variable x. Which of the two intervals is wider? (9,9)

6. (a) Suppose we postulate the model $E(Y) = \beta_0 + \beta_1 x$ but the model $E(Y) = \beta_0 + \beta_1 x + \beta_{11} x^2$ is the true response function, unknown to us. If we use observations of Y at $x = -1, 0, 1$ to estimate β_0 and β_1 in the postulated model, what biases will be introduced?

(b) Develop a suitable test for testing the significance of regression in multiple linear regression $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$. Further, show that the test statistics is equivalent to

$$F_0 = \frac{R^2(n-k-1)}{(1-R^2)k}.$$

(c) Suppose that a linear regression model with $k = 2$ regressors has been fitted to $n = 25$ observations and $R^2 = 0.90$. Use the results of the previous problem to test for significance of regression at $\alpha = 0.05$ (Given that $F_{0.05}[2,22] = 3.44$). What is the smallest value of R^2 that would lead to the conclusion of a significant regression if $\alpha = 0.05$? (6,8,4)

7. Write short notes on the following :
(a) Residual Analysis
(b) Technique of Analysis of Covariance
(c) Orthogonal Polynomials (6,6,6)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1106 **I**

Unique Paper Code : 2372013502

Name of the Paper : Linear Models

Name of the Course : **B.Sc. (Hons.) Statistics under NEP-UGCF**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer **five** questions in all. Q. No. 1 is compulsory.
3. **All** questions carry equal marks.
4. Show all your calculations clearly.
5. Use of simple scientific calculators is allowed.

1. (a) Define a statistical linear model and distinguish between fixed and random effects linear models.

(b) What is a linear parametric function? When is it said to be estimable?

(c) Let X_i, Y_i, Z_i, $i = 1, 2$ be independent observations with common variance σ^2. If $E(X_i) = \theta_1$, $E(Y_i) = \theta_2$, $E(Z_i) = \theta_1 - \theta_2$, $i = 1, 2$.

(i) Find the BLUEs of θ_1, θ_2 and $\theta_1 + \theta_2$.

(ii) Obtain the $V(\hat{\theta}_1), V(\hat{\theta}_2)$ and $\text{cov}(\hat{\theta}_1, \hat{\theta}_2)$.

(d) Consider a simple linear regression model $y = \alpha + \beta x + \varepsilon$ where α and β are unknown parameters and ε is random error with mean zero. Based on 10 independent observations (x_i, y_i); $i = 1,2, \ldots,10$, the fitted model, using OLSE. is $\hat{y}_i = 1.5 + 0.8x_i$. If $S_{yy} = 5$ and $S_{xx} = 6$, what is the value of the adjusted coefficient of determination? (4.5,4.5,4.5,4.5)

2. For a given model $\underset{\sim}{Y}_{n\times 1} = X_{n\times p}\underset{\sim}{\beta}_{p\times 1} + \underset{\sim}{\varepsilon}_{n\times 1}$ with $E(\underset{\sim}{\varepsilon}) = \underset{\sim}{0}, V(\underset{\sim}{\varepsilon}) = \sigma^2 I$ and $\rho(X) = p < n$, prove that the least squares estimator of $\underset{\sim}{\beta}$ is BLUE. Also, obtain an unbiased estimator of σ^2. (18)

3. (a) Consider a quadratic form $Q = \underset{\sim}{Y}'A\underset{\sim}{Y}$, where A is a symmetric matrix of order n × n and $\underset{\sim}{Y}_{n\times 1}$ is *NID* $(\underset{\sim}{0}, I_n)$, then prove that a necessary and sufficient condition for the quadratic form Q to be distributed as chi-squared variate with k d.f is that A is an idempotent matrix of rank k.

(b) To test the effectiveness of three different teaching methods, three instructors were assigned 12 students each. Equal number of students were then randomly assigned to the different teaching

methods and were taught the same material. The following ANQVA table was obtained for the grades of students who were given identical tests:

Source	DF	SS	MS	F-ratio
Methods		162		
Instructors		90		
Interaction		600		
Error		900		
Total				

Complete all the missing entries in the table above. (12,6)

4. Derive the analysis of variance for one-way classified data under fixed effect model. Also, obtain the variance of the estimated parameters. How would you proceed to test the hypothesis of equality of two specific class means? (18)

5. (a) Suppose that we are fitting a straight line and wish to make the standard error of the slope as small as possible. If the "region of interest" for x is $-1 \le x \le 1$, then where should the observations $x_1, x_2, \ldots, x_n$ be taken when (i) n is even and (ii) n is odd?

(b) Develop for the simple linear regression model with intercept a confidence interval on the mean response at x_0 and a prediction interval for the