

(c) Fisher-Neyman criterion

(d) Sufficient conditions for consistency (15)

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(1000)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1068 I

Unique Paper Code : 2372013501

Name of the Paper : Theory of Estimation

Name of the Course : B. Sc. (Hons.), Statistics
under NEP-UGCF

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt six questions in all, selecting three questions from **section I** and three questions from **section II**.
3. Use of simple calculator is allowed.

P.T.O.

SECTION I

1. (a) State and prove Invariance property of consistent estimators. Using this property, obtain consistent estimator of $\theta^2 + \theta - \sqrt{\theta}$, when X follows Bernoulli distribution with parameter θ .

- (b) Let $X_1, X_2, \dots, X_n$, be a random sample from population having p.d.f. $f(x, \theta) = (\theta+1)x^\theta$,

$0 < x < 1, \theta > -1$. Show that $\left[\frac{-(n-1)}{\sum_{i=1}^n \log x_i} - 1 \right]$ is an

unbiased estimator of θ (8, 7)

2. (a) Show that the estimator of the form $a\bar{X}$ for in random sampling from $N(\theta, \sigma^2)$, has the minimum

(i) MLE may not be unbiased

(ii) MLE may not be unique (8, 7)

7. (a) Let $X_1, X_2, \dots, X_n$, be a random sample from a distribution having p.d.f. $f(x, \theta) = e^{-(x-\theta)}$, $\theta \leq x < \infty, -\infty < \theta < \infty$. Obtain 100 $(1-\alpha)\%$ confidence interval for θ .

- (b) Describe a method of constructing confidence intervals. Find 100 $(1-\alpha)\%$ confidence interval for binomial proportion based on a random sample of size n for large samples. (7, 8)

8. Describe any three of the following:

(a) Minimum Chi-square method of estimation

(b) Rao-Blackwell theorem

(b) Let $X_1, X_2, \dots, X_n$, be a random sample from a

distribution with p.d.f. $f(x; \theta) = \frac{1}{\theta} x^{\frac{1-\theta}{\theta}}, 0 < x < 1, 0$

$< \theta < \infty$. Find the Maximum likelihood estimator of θ . Also, examine the estimator for unbiasedness property. (8,7)

6. (a) What is failure-censored sample in life testing experiment? Obtain the Maximum likelihood estimator of the expected life time and reliability function in case of failure censored sample from the life time distribution:

$$f(x; \sigma) = \frac{1}{\sigma} e^{-\frac{x}{\sigma}}, x > 0, \sigma > 0.$$

(b) Show by means of an example that

mean-square error, when $a = \frac{\theta^2}{(\theta^2 + \frac{\sigma^2}{n})}$, Which

$\rightarrow 1$ as $n \rightarrow \infty$ but < 1 when n is finite.

(b) State Cramer-Rao Inequality. Let $X_1, X_2, \dots, X_n$, be a random sample from Population having p.d.f.

$$f(x, \theta) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x < \infty, -\infty < \theta < \infty.$$

Find Cramer-Rao Lower bound for variance of an unbiased estimator of θ . Also, examine whether MVB estimator exists for θ . [7, 8]

3. (a) Let X and Y be two random variables having joint

probability density function $f(x, y) = \frac{2}{\theta^2} \exp[-(x+y)$

$/\theta], 0 < x < y < \infty$.

(i) Show that the mean and variance of Y are

respectively $\frac{3\theta}{2}$ and $\frac{5\theta^2}{4}$.

(ii) Show that $E[Y|X = x] = \phi(x) = x + \theta$ and expected value of $X + \theta$ is that of Y .

(iii) Show that the variance of $\phi(X)$ is less than that of Y .

(iv) Comment on the result.

(b) Define Minimum Variance Unbiased (MVU)

Estimator. Let T_0 be an MVU estimator, while T_1

is an unbiased estimator with efficiency e_θ . If ρ_θ ,

be the correlation coefficient between T_0 and T_1 ,

then prove that $\rho_\theta = \sqrt{e_\theta}$. (8, 7)

4. (a) Let $X_1, X_2, \dots, X_n$, be a random sample from $U(0, \theta)$ population. Show that the largest order statistic $X_{(n)}$ is a complete sufficient statistic for θ . Using Lehman Scheffe's theorem, find UMVUE of θ .

(b) Let $X_1, X_2, \dots, X_n$, be a random sample from the

distribution having p.d.f. $f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad 0 \leq x$

$< \infty, \theta > 0$. Show that $\frac{n-1}{n\bar{X}}$ is the only unbiased

estimator of $\frac{1}{\theta}$ based on $\bar{X}$. [8,7]

SECTION II

5. (a) Describe the Method of Moments. Let $X_1, X_2, \dots, X_n$, be a random sample from the distribution with

p.d.f. $f(x; \alpha, \beta) = \frac{\beta^n}{\Gamma \alpha} x^{n-1} e^{-\beta x}, x \geq 0, \alpha > 0, \beta > 0$.

Estimate the parameters α and β by the Method of Moments.