(b) Prove that the systematic sampling will be more efficient than SRSWOR if the intra class correlation coefficient $\rho < -\frac{1}{N-1}$. (Notations have their usual meaning). (8,7)

8. Write short note :

(i) Stratified Random Sampling

(ii) Probability and non-probability sampling

(iii) Steps in conducting sample surveys (5×3)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1087 **I**

Unique Paper Code : 2372012301 (NEP-UGCF)

Name of the Paper : Sample Surveys (DSC)

Name of the Course : **B.Sc. (Hons.) STATISTICS**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

1. (a) Define Simple random sampling. Derive the variance of the sample mean based on SRSWOR. Also, derive the unbiased estimator of this variance.

(b) From a simple random sample of size n drawn from N units by SRSWOR, a simple random sub-sample of ni units is duplicated and added to the original sample. Show that the mean based on $(n + n_1)$ units is an unbiased estimator of the population mean. Also, obtain its variance. How does it compare with the variance of the estimator based on n units only? (7,8)

2. (a) If (X_i, Y_i) is the pair of the variables defined for every unit (i = 1,2,...,N) of the population and $\bar{y}_n$ and $\bar{x}_n$ are the corresponding sample means of a

(b) Define Difference estimator and derive from it the regression estimator. Also obtain the variance of regression estimator under first approximation. (6,9)

7. (a) For a given cost function $C = c_1n_1 + c_2n_2$, derive $Var(\bar{y}_{st})$ under optimum allocation. Further, assume that a sampler has two strata with relative sizes P_1 and P_2 and equal population standard deviations S_1 and S_2. He prefers to use proportional allocation but does not wish to incur a substantial increase in variance as compared with optimum allocation. Show that (ignoring fpc):

$$\frac{Var(\bar{y}_{st})_{prop}}{Var(\bar{y}_{st})_{opt}} = \frac{P_1c_1 + P_2c_2}{\left(P_1\sqrt{c_1} + P_2\sqrt{c_2}\right)^2}$$

Compute the sample sizes n_1, n_2 in the 2 strata minimizing the total size of sample in each case under the following conditions (ignore the fpc):

(i) The standard error of the estimated population mean is to be 0.1.

(ii) The standard error of the difference between the two estimated stratum means is to be 0.1.

(iii) The standard error of the estimated mean of each stratum is to be 0.1. (6,9)

6. (a) A population of N units contains N_1 units belonging to class A and N_2 $(N-N_1)$ units belonging to class B. Obtain an unbiased estimator of N_1 and unbiased estimator of its variance, under simple random sampling without replacement.

simple random sample of size n selected without replacement, then prove that

$$\operatorname{cov}(\bar{x}_n, \bar{y}_n) = \frac{N-n}{n(N-1)}\operatorname{cov}(x,y).$$

Hence prove that $r(\bar{x}_n, \bar{y}_n) = \rho(X,Y)$. (Notations have their usual meaning).

(b) Derive the condition under which a systematic sampling has the same precision as the corresponding stratified random sampling with one unit per stratum. (8,7)

3. (a) Compare stratified random sampling under Neyman and proportional allocations with simple random sampling without replacement.

(b) State the practical difficulties in adopting Neyman method of allocation of a sample to different strata. How much would be the increase in variance, on the average, if the allocation is based on the estimates of strata mean squares? Also, compare it with proportional allocation. (6,9)

4. (a) (i) In SRSWOR, prove that the probability of selecting a specified unit of the population at any given draw is equal to the probability of selecting it at the first draw.

(ii) If a random sample of size n is drawn from a population of size N by SRSWOR, then prove that covariance between any two members of the sample is $-\frac{\sigma^2}{N-1}$.

(b) Obtain the bias in the ratio estimator to the first approximation. Also show that this bias vanishes if the regression line is linear and passes through origin. (3,4,8)

5. (a) Explain the Yates' end corrections in systematic sampling for a population with a linear trend. Hence show that the sample mean obtained after applying the end corrections overlaps the population mean.

(b) In a stratification with 2 strata, the values of the P_i and S_i are as follows :

Stratum	P_i	S_i
1	0.8	2
2	0.2	4