

ca implies $b = c$ for all a, b, c in G then G is Abelian (that is, left-right cancellation property implies Abelian). (2+2.5+3)

(c) Show that the set $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R}, a \neq 0 \right\}$

is a group under matrix multiplication. (7.5)

6. (a) State two-step subgroup test. Let G be an Abelian group and H, K be subgroups of G then show that

$$HK = \{hk \mid h \in H, k \in K\}$$

is a subgroup of G . (2+5.5)

- (b) Define order of an element ' a ', $O(a)$, in a group G . Prove that in any group G , $O(bab^{-1}) = O(a)$ for all $a, b \in G$. (2+5.5)

- (c) Write all the generators of the cyclic group Z_{24} . Further describe all the subgroups of Z_{24} and find all generators of the subgroup of order 8 in Z_{24} . (3+3+1.5)

(1000)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1363

I

Unique Paper Code : 2352011101

Name of the Paper : Algebra (DSC-1)

Name of the Course : B.Sc. (H) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
 2. Attempt all questions by selecting **two** parts from each question.
 3. **All** questions carry equal marks.
1. (a) (i) Find a cubic equation with real coefficients two of whose roots are 1 and $3+2i$. Also state the result being used.

P.T.O.

- (ii) Find an upper limit (using both the theorems) to the roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0. \quad (3.5+4)$$

- (b) Solve $3x^3 + 11x^2 + 12x + 4 = 0$, being given that roots are in Harmonic progression. (7.5)

- (c) Find all the rational roots of $6y^3 - 11y^2 + 6y - 1 = 0$. (7.5)

2. (a) Compute $z^n + \frac{1}{z^n}$ if $z + \frac{1}{z} = \sqrt{3}$. (7.5)

- (b) Find $|z|$, $\arg z$, $\text{Arg } z$, $\arg(-z)$ for $z = (7-7\sqrt{3} - i)$ (7.5)

- (c) Solve the equation $z^7 - 2iz^4 - iz^3 - 2 = 0$. (7.5)

3. (a) Solve $28x^3 + 9x^2 - 1 = 0$ by Cardan's method. (7.5)

- (b) If a , b and c are non-zero integers with a and c relatively prime, prove that $\gcd(a, bc) = \gcd(a, b)$ (7.5)

- (c) (i) Find $\gcd$ of 1800 and 756 and express it in the form $ma + nb$ for some integers m and n .

- (ii) If a and b are relatively prime integers, prove that $\gcd(a + b, a - b) = 1$ or 2 . (4+3.5)

4. (a) Solve the following pair of congruences, if possible. If no solution exists, explain why not.

$$2x + y \equiv 1 \pmod{6}$$

$$x + 3y \equiv 3 \pmod{6}$$

(7.5)

- (b) If $a \equiv x \pmod{n}$ and $b \equiv y \pmod{n}$, then

$$(i) \quad a + b \equiv x + y \pmod{n}$$

$$(ii) \quad ab \equiv xy \pmod{n} \quad (3.5+4)$$

- (c) State fundamental theorem of arithmetic. Suppose a and b are integers and p is a prime such that $p \mid ab$. Prove that $p \mid a$ or $p \mid b$. (2.5+5)

5. (a) Describe symmetries of a non-square rectangle with diagrams. Also, construct the corresponding Cayley table. (3.5+4)

- (b) Define an Abelian group. Show that in a group G if $ab = ac$ then $b = c$ (called left cancellation property). Further, show that in a group G if $ab =$

P.T.O.