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S. No. of Question Paper : 2315

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor

Types of the Paper : DSC-B2

Semester : III

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts from each question.

All questions carry equal marks.

Use of simple calculator is allowed.

1. (a) Find the differential equation of the family of curves $y = e^x (A \sin x + B \cos x)$, where A and B are arbitrary constants. 7.5

- (b) Solve the initial value problem :

$$(x^2 + 1) \frac{dy}{dx} + 4xy = x, y(2) = 1.$$

- (c) Solve the differential equation :

7.5

$$x(1 + y^2) dx + y(1 + x^2) dy = 0.$$

7.5

P.T.O.

2. (a) In a certain bacteria culture, the rate of increase in the number of bacteria is proportional to the number present at any time. 7.5

(i) If the number triples in 5 hours, how many will be present in 10 hours ?

(ii) When will the number present be 10 times the number initially present ?

- (b) Show that $y_1 = x^3$ and $y_2 = x^{-2}$ are solutions of the differential equation

$$x^2 y'' - 6y = 0.$$

Also, verify that y_1 and y_2 are linearly independent. 7.5

- (c) Find the integrating factor of the following differential equation and solve it

$$(y - xy)dx + x dy = 0. \quad 7.5$$

3. (a) Find the general solution of the differential equation by using the method of undetermined coefficients 7.5

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = x^2 e^{2x}.$$

- (b) Solve the initial value problem : 7.5

$$y'' - 4y' + 3y = 0, y(0) = 6, y'(0) = 7.$$

- (c) Find the general solution of the differential equation by using the method of variation of parameter :

$$y'' + y' - 6y = \sin(2x). \quad 7.5$$

4. (a) Find the general solution of the given differential equation : 7.5

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} - 3y = 4x.$$

- (b) Write the general solution of the 10th-order homogeneous differential equation whose auxiliary equation has the following roots : 7.5

$$3, 3, 3, -3, 5 + i, 5 - i, 0, 0, 1 + i, 1 - i.$$

Verify that e^{2x} and e^{3x} are solutions to the homogeneous differential equation

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0.$$

Show that a linear combination of e^{2x} and e^{3x} is also a solution.

- (c) Find the general solution of the linear system : 7.5

$$\frac{dx}{dt} + \frac{dy}{dt} - x = -2t,$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t^2.$$

5. (a) Eliminate the arbitrary constants a and b from the equation :

$$2z = (ax + y)^2 + b,$$

and obtain the partial differential equation. Also, specify the order. 7.5

- (b) Find the solution of the Cauchy problem for the first order PDE :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2xy, \text{ with the condition } u = 2 \text{ on } y = x^2. \quad 7.5$$

- (c) Reduce the equation :

$$yu_x + u_y = x$$

to canonical form and obtain the general solution.

7.5

6. (a) Use the method of characteristics to find the general solution of the equation :

$$xu_x + yu_y = xy.$$

7.5

- (b) Apply the method of separation of variables $u(x, y) = X(x) Y(y)$ to solve

$$u_x - u_y = 0, u(0, y) = 4e^{3y}.$$

7.5

- (c) Classify the partial differential equation as elliptic, parabolic or hyperbolic :

$$yu_{xx} + 3yu_{xy} + 3u_x = 0, y \neq 0.$$

Also, reduce it to canonical form and obtain the general solution.

7.5