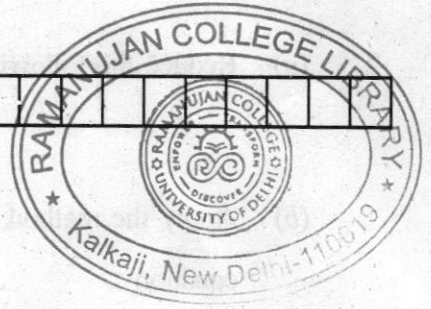


This question paper contains 4 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--



S. No. of Question Paper : 1121

Unique Paper Code : 2352013503

Name of the Paper : Partial Differential Equations

Name of the Course : Bachelor of Science (Honours Course) Mathematics

Semester : V

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Use of calculator is not allowed.

1. (a) Define the linear and non-linear partial differential equations with examples. Find the first order partial differential equation satisfied by the family of right circular cones whose axes coincide with the z -axis, and is given by :

$$x^2 + y^2 = (z - c)^2 \tan^2 \alpha.$$

- (b) Solve the following initial value problem using method of characteristics :

$$u_t + 2uu_x = v - x, \quad v_t - cv_x = 0 \quad \text{with} \quad u(x, 0) = x, \quad v(x, 0) = x.$$

- (c) Solve the equation $u_x + xu_y = y$ with the Cauchy data $u(1, y) = 2y$.

P.T.O.

2. (a) Reduce the following equation into canonical form and then find the general solution :

$$u_x + 2xyu_y = x.$$

- (b) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve the following equation :

$$u_x + 2u_y = 0, \quad u(0, y) = 3e^{-2y}.$$

- (c) Find a complete integral of the equation by using Charpit's method :

$$p = (z + qy)^2.$$

3. (a) Show that the equation of motion of the vibrating string is :

$$u_{tt} = c^2 u_{xx}, \quad c^2 = T/\rho,$$

where T is the tension at the end point of the string and ρ is the density.

- (b) Classify and determine the region in which the following equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form :

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0.$$

- (c) Find the traffic density $\rho(x, t)$, satisfying :

$$\frac{\partial \rho}{\partial t} + x \sin(t) \frac{\partial \rho}{\partial x} = 0,$$

with the initial condition $\rho_0(x) = 1 + \frac{1}{1+x^2}$.

4. (a) Transform the following equation to the form $u_{\xi\eta} = cv$, $c = \text{constant}$,

$$3u_{xx} + 7u_{xy} + 2u_{yy} + u_y + u = 0,$$

by introducing the new variables $v = ue^{a\xi + b\eta}$, where a and b are undetermined coefficients.

(3)

- (b) Define the homogeneous and non-homogeneous of partial differential equations with examples, and find the general solution of the equation :

$$u_{xxxx} - u_{yyyy} = 0.$$

- (c) Find the general solution of the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y + e^{2x+y}.$$

5. (a) Determine the solution of the initial-value problem for the semi-infinite string with a fixed end :

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= f(x), u_t(x, 0) = g(x), & 0 \leq x < \infty \\ u(0, t) &= 0, & 0 \leq t < \infty \end{aligned}$$

- (b) Determine the solution of the initial-value problem for the semi-infinite string with free end :

$$\begin{aligned} u_{tt} &= 9u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= 0, u_t(x, 0) = x, & 0 \leq x < \infty \\ u_x(0, t) &= 0, & 0 \leq t < \infty \end{aligned}$$

- (c) Determine the solution of the initial-value problem with non-homogeneous boundary conditions :

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= \sin x, u_t(x, 0) = 0, & 0 \leq x < \infty \\ u(0, t) &= x, & 0 \leq t < \infty \end{aligned}$$

(4)

1121

6. (a) Determine the solution of the Cauchy problem for non-homogeneous wave equation :

$$u_{tt} - c^2 u_{xx} - \sin x = 0, \quad u(x, 0) = \cos x, \quad u_t(x, 0) = 1 + x.$$

- (b) Determine the solution of the initial-value problem :

$$u_{tt} = 16u_{xx},$$

$$u(x, 0) = 1 + x, \quad u_t(x, 0) = x^3, \quad 0 < x < \infty, t > 0$$

$$u_x(0, t) = \cos x, \quad 0 \leq x < \infty$$

$$0 \leq t < \infty$$

- (c) Determine the solution of the initial-value problem :

$$u_{tt} - c^2 u_{xx} = 0$$

$$u(x, 0) = \log(1+x^2), \quad u_t(x, 0) = 2, \quad 0 < x < \infty, t > 0$$

$$0 \leq x < \infty$$