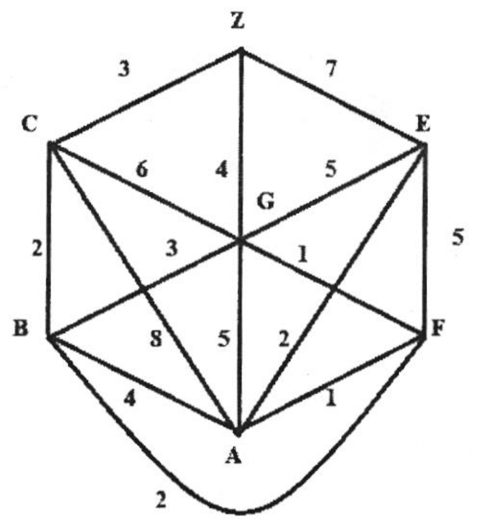
(c) Use improved Dijkstra's Algorithm to find the shortest path between A and Z for the graph given below. Write the steps



(6½)

26/12/24 (M)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6086 **I**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. All the given six questions are compulsory to attempt.
3. Do any two parts from each of the given six questions.
4. Marks for each part are indicated on the right in brackets

P.T.O.

SECTION I

1. (a) If '1', '2','3' denote chains of one, two, three elements respectively, then draw Hasse diagrams for:

   (i) $L \times K$, when $L = 3 \times 3$ and $K = 2$

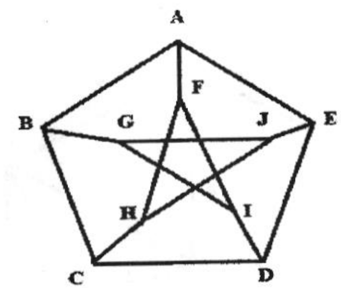
   (ii) $L \oplus K$, when $L = 1$ and $K = 2 \times 2$ (6)

   (b) Let P, Q and R be ordered sets and let $\varphi: P \rightarrow Q$ and $\psi: Q \rightarrow R$ be order preserving maps. Then show that the composite map: $\psi o \varphi: P \rightarrow R$ given by:

   $(\psi o \varphi)(x) = \psi(\varphi(x))$ for $x \in P$, is also an order preserving map (6)

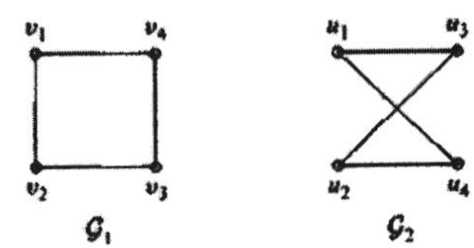
   (c) Consider the diagram below of the ordered subset $P = \{1, 2, 3, 4, 5, 6, 7\}$ of $(N_0; \leq)$, where $(N_0; \leq)$ is the ordered set of non-negative integers ordered by relation $\leq$ on N_0, defined as: For m, n $\in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \in N_0$: $n=km$.

6. (a) Define Hamiltonian circuit in a graph. Is the graph given below Hamiltonian? Explain your answer.



(6½)

   (b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 (with vertices v_1, v_2, v_3, v_4) and G_2 (with vertices u_1, u_2, u_3, u_4)shown below. Find a permutation matrix P such that: $A_2 = PA_1P^T$



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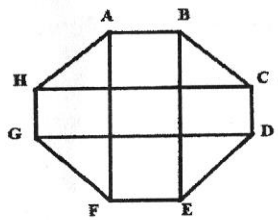
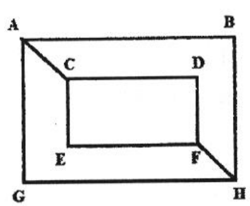


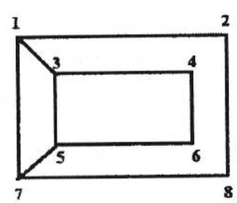
(ii) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 4? (6)

(b) (i) Does a graph with degree sequence **5, 4, 4, 3, 2, 1** exist? Justify your answer.

(ii) Draw K_5 and $K_{3,4}$ (6)

(c) Define Isomorphism of graphs. Determine whether the following graphs are isomorphic or not? Give explanation for your answer.





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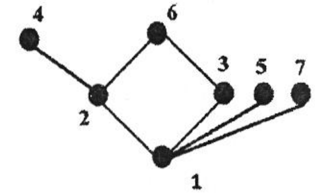

Then, for the following subsets of P, find the following meet and join as indicated. Either specify the meet and join, if they exist or indicate why they fail to exist.

(i) meet and join of subset {3,5,7}

(ii) meet and join of subset {2,3,6} (6)

2. (a) Define a Complete Lattice. Show that for any set X, the ordered set $(\wp(X), \subseteq)$ is a complete lattice, where $\wp(X)$ denotes the power set of X and $\subseteq$ is the inclusion relation. (6½)

(b) For a lattice L with operations $\vee$ and $\wedge$, show that $\forall\, a, b, c \in L$,

(i) $a \vee (a \wedge b) = a$

(ii) $(a \vee b) \vee c = a \vee (b \vee c)$ (6½)

(c) Let L be a lattice. Then show that $\forall\, a, b, c, d \in L$:

$a \leq b$ and $c \leq d$ implies $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$ (6½)

SECTION II

3. (a) Define a modular lattice. Show that the cartesian product of two Modular lattices is a Modular lattice. (6)

(b) State $M_3 - N_5$ theorem and use it to show that every chain is distributive lattice. (6)

(c) Define the conjunctive normal form and disjunctive normal form of a Boolean polynomial. Find disjunctive normal for the Boolean polynomial given below:

$(x + y')(y + z')(z + x')(x' + y')$ (6)

4. (a) Use the Quine-McClusky method to find minimal form of polynomial below:

$wx'y'z + w'xy'z' + wx'y'z' + w'xyz + w'x'y'z' + wxyz + wx'yz + w'xyz' + w'x'yz'$ (6½)

(b) Find the minimal form using the Karnaugh diagram for the following Boolean polynomial

$x_3(x_2 + x_4) + x_2x_4' + x_2'x_3'x_4$ (6½)

(c) Define complemented lattice and prove that for all x, y in a Boolean Algebra,

$(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ (6½)

SECTION III

5. (a) (i) Define bipartite graph. Determine whether the given graph is complete bipartite graph or not? Justify your answer.