(b) What will be the average number of persons affected with Pneumonia in 3 weeks duration if $\lambda = 8$ and $p = 0.9$? (8,7)

7. (a) If the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $1/\lambda$, then show that the event E forms a Poisson Process with mean λt.

(b) Describe the classical ruin problem. Find the probability of the gambler's ultimate ruin. (8,7)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **1125** **I**

Unique Paper Code : 2372013503 (NEP-UGCF)

Name of the Paper : Stochastic Processes

Name of the Course : **B.Sc. (Hons.)**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **TWO** sections in this paper.
3. **All** sections are compulsory.
4. Use of nonprogrammable scientific calculator is allowed.

Section I

1. Attempt any **FIVE** parts from following :

(b) Obtain the generating function of the Fibonacci numbers $\{F_n\}$ defined by

$$F_n = F_{n-1} + F_{n-2},\ n \geq 2,\ F_0 = 0,\ F_1 = 1.$$

(c) If X and Y are nonnegative integral valued independent random variables with probability generating functions A(s) and B(s) respectively, obtain the generating function of $W = X - Y$. Comment on the values of W.

(d) Let $\{X_n, n \geq 1\}$ be a sequence of uncorrelated random variables with mean 0 and variance 1. Is $\{X_n, n \geq 1\}$ a covariance stationary process?

(e) Six boys Dev (D), Hemant (H), Jeet (J), M (Mohan), S (Sunil) and T (Tejas) play catch. If D has the ball, he is equally likely to throw it to H, M, S and T. If H gets the ball, he is equally likely to throw it to D, J, S, T. If S has the ball, he is

(iii) $\mathrm{Cov}\{X_{n-1}, X_n\}$

(iv) Further, what is $\mathrm{Cov}\{X_{n-1}, X_n\}$ for the case $a = b$?

(b) Let $(X_n, n \geq 0\}$ be a Markov chain having state space $S = \{0, 1, 2, 3\}$ transition matrix as :

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.3 & 0.2 & 0.5 & 0 \\ 0 & 0.2 & 0.3 & 0.5 \end{bmatrix}$$

(i) Is the chain irreducible?

(ii) Find the nature of all states. (8,7)

6. Let the number of persons taking a particular vaccine for Flu follow Poisson Process with parameter λ per week. A vaccinated person has probability p of not developing serious pneumonia (independent of others who received the vaccine).

(a) Derive the probability distribution of number of individuals who can get Pneumonia even after taking the vaccine.

(i) $P(X_1 = 1) = P(X_2 = 1) = P(X_3 = 1)$.

(ii) $P(X_3 = 1|X_2 = 1) \neq (X_3 = 1|x_2 = 1, X_1 = k)$ for $k = 0, 1$.

(iii) Define a Markov Chain. Does $\{X_n\}$ under *Polya's Urn Model* constitutes a Markov Chain? (8,7)

5. (a) Consider a Markov chain $(X_n, n \geq 0)$ such that X_n assumes values -1 and 1 with conditional probabilities :

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0 < a,\ b < 1$$

and initial distribution $P\{X_0 = 1\} = p_1 = 1 - P\{X_0 = -1\}$.

Defining $P\{X_n = 1\} = p_n$, obtain the expressions for

(i) $E\{X_n\}$

(ii) $Var\{X_n\}$

equally likely to throw it to D, H, M and T. If either J or T gets the ball, they keep throwing it to each other. If M gets the ball, he runs away with it. Obtain the transition probability matrix.

(f) Show that $\frac{N(t)}{t}$ is a reasonable (consistent) estimate of the mean rate λ for the Poisson process $\{N(t),\ t \geq 0)$.

(g) Describe the classical random walk problem associated with a particle which starts at a position z and the walk is confined between points 0 and a. (5×3=15)

Section II

(Attempt Any **FIVE** *Questions.)*

2. (a) Let $S_N = X_1 + X_2 + \cdots + X_N$, where X_i's are i.i.d. random variables and N is a random variable independent of X_i's. Show that

$$\operatorname{Var}(S_N) = E(N)\operatorname{Var}(X) + \operatorname{Var}(N)[E(X)]^2 \text{ and}$$

$$\frac{\operatorname{Var}(S_N)}{E(S_N)} = \frac{\operatorname{Var}(X)}{E(X)} + \frac{E(X)\operatorname{Var}(N)}{E(N)}.$$

(b) Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Show that the generating function of X is $\left(\frac{s^2}{4}\right)\left(1-\frac{s}{2}-\frac{s^2}{4}\right)^{-1}$.

(8,7)

3. (a) Let X have a zero truncated Poisson distribution with

$$P(X=k) = \frac{a^k}{(e^a-1)k!};\ k=1,2,3,\ldots$$

Determine the following :

(i) Probability generating function P(s) of X

(ii) E(X)

(iii) var(X)

(iv) Verify that P(1) = 1.

(b) Define the following :

(i) Stochastic processes

(ii) Covariance Stationary Process

(iii) Closed Set

(iv) Ergoidic State (8,7)

4. (a) Consider a sequence $\{X_n, n \geq 2\}$ of independent coin-tossing trials with probability p for head (H) in a trial. Denote the states of X_n by states 1, 2, 3, 4, according to the trial numbers (n – 1) and n results in HH, HT, TH, TT respectively. Show that $\{X_n, n \geq 2\}$ is a Markov chain. Find the transition probability matrix P. Show that P^m ($m \geq 2$) is a matrix with all the four rows having the same elements.

(b) Describe the general Polya's Urn model for b black and r red balls. Under this model, consider $X_n = 1$ which denotes the event that the n-th ball drawn is black ($X_n = 0$ for red ball). Show that