

This question paper contains 8 printed pages].

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S. No. of Question Paper : 1312

Unique Paper Code : 2353012001

Name of the Paper : Graph Theory

Type of Paper : DSE

Name of the Course : B.Sc. (H) Mathematics

Semester : III

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions has *three* parts (a), (b) and (c). You have to attempt any *two* parts of each question.

All questions carry equal marks.

Parts of each question to be attempted together.

Use of calculator is not allowed.

1. (a) (i) Draw a graph with 5 vertices and as many edges as possible. How many edges does your graph contain ? What is the name of this graph and how is it denoted ?

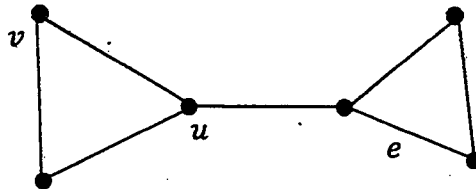
(ii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4.

Justify your answer.

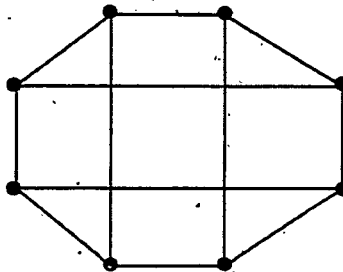
4.5,3

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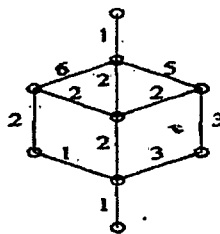
- (b) (i) Define sub-graph of a graph. Draw pictures of the subgraphs $G \setminus \{e\}$, $G \setminus \{v\}$ and $G \setminus \{u\}$ of the following graph G :



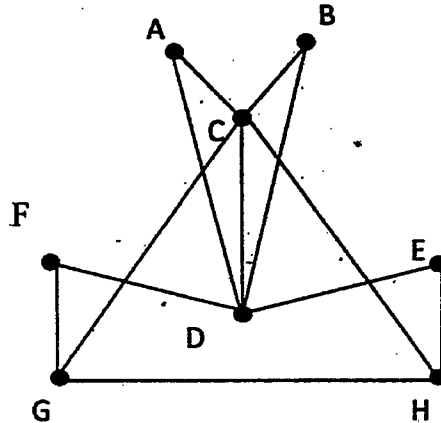
- (ii) What is bipartite graph ? Determine whether graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite : 4,3.5



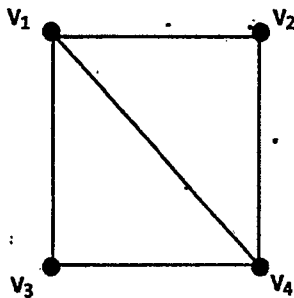
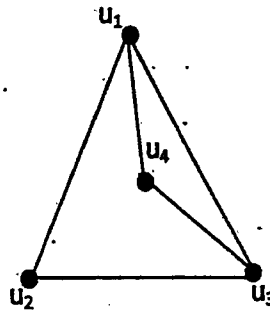
- (c) (i) Draw all non-isomorphic graphs for $n = 3$ vertices. Give the degree sequence of each graph.
- (ii) Solve the Chinese Postman Problem for the graph below : 4,3.5



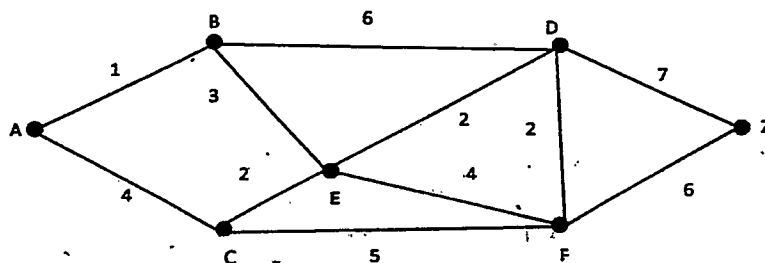
2. (a) Define Eulerian graph and Hamiltonian graph. Consider the graph given below. Is it Hamiltonian ? If no, explain your answers, if yes find a Hamiltonian cycle. Also check whether the graph is Eulerian or not : 7.5



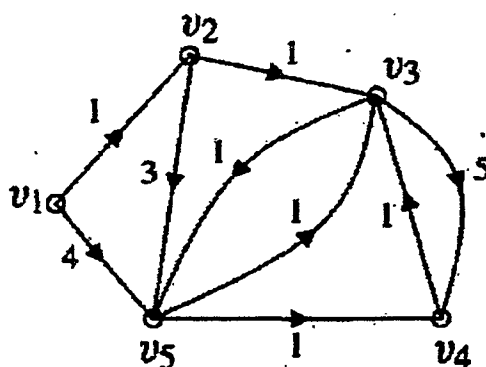
- (b) Define adjacency matrix of a graph. Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$: 7.5

 G_1  G_2

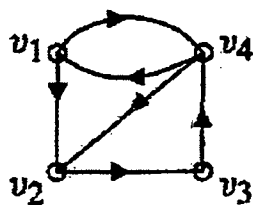
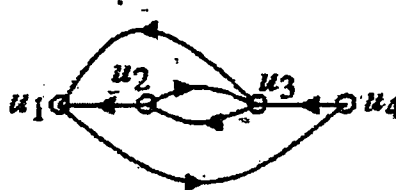
- (c) Apply the Dijkstra's Algorithm to find a shortest path from A to Z in the graph shown. Label all vertices and write steps : 7.5



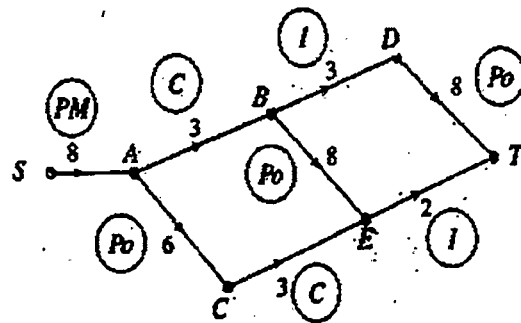
3. (a) Apply the Bellman-Ford algorithm to find the shortest distance from v_1 to all other vertices in the following Weighted Digraph : 7.5



- (b) When are two Digraphs said to be isomorphic ? Are the following Digraphs G_1 and G_2 isomorphic ? Explain your answer : 7.5

 G_1  G_2

- (c) The construction of a certain part in an automobile engine involves four activities : pouring the mold, calibration, polishing and inspection. The mold is poured first; calibration must occur before inspection. Pouring the mold takes 8 units of time, calibration takes 3 units, polishing takes 6 units for an uncalibrated product and 8 units for a calibrated one, and inspection takes 2 units of time for a polished product and 3 units for an unpolished one :



- (i) What type of scheduling problem is this (type I or II) ? Why ?
 (ii) What is the shortest time required for this job ? Describe the critical path.

2, 5.5

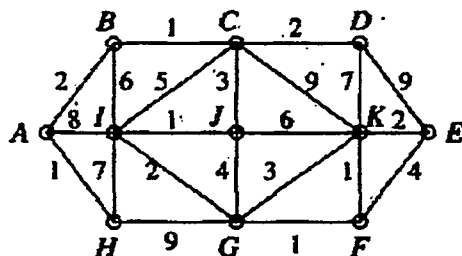
4. (a) (i) Define tree and find the number of spanning trees in $K_{2,2}$.

- (ii) A connected graph with n vertices is a tree if and only if it has $n - 1$ edges.

4, 3.5

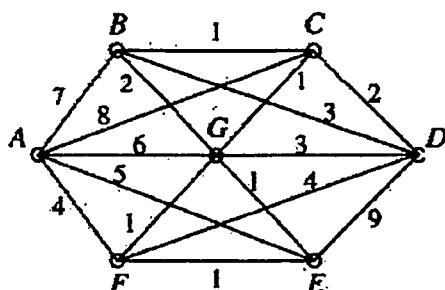
P.T.O.

- (b) Find a minimum spanning tree for the graph using Kruskal algorithm. Give the weight of your minimum tree and show your steps : 7.5

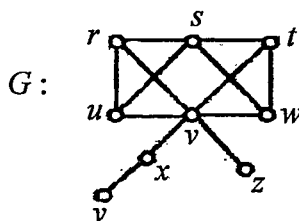


- (c) (i) Find a Hamiltonian cycle of lowest weight. What is the weight ?
 (ii) By removing vertex E, find a lower bound for the weight of any Hamiltonian cycle.
 (iii) By removing vertex G, find a lower bound for the weight of any Hamiltonian cycle.

2.5, 2.5, 2.5



5. (a) (i) Define cut-vertex of a Graph G and find all the cut-vertices of the graph G given below :



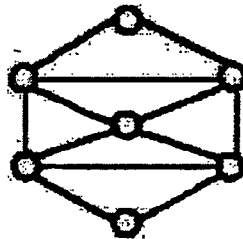
- (ii) Draw the induced graph by deleting all the cut-vertices.

4.5, 3

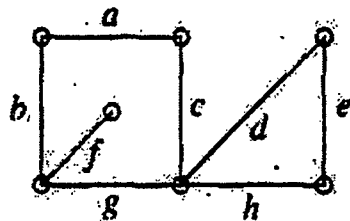
(b) When can we say a bipartite graph satisfies Hall's condition and show that every r -regular bipartite graph ($r \geq 1$) has a perfect matching. 7.5

(c) (i) Prove that for every positive integer, $\lambda(K_n) = n - 1$.

(ii) Find the $k(G)$, $\lambda(G)$ and $\delta(G)$ for the below graph G with explanation : 4,3,5



6. (a) Consider the planar graph given below :



(i) How many regions are there ? Mention it.

(ii) List the edges which form the boundary of each region.

(iii) Which region is exterior ?

3,3,1.5

P.T.O.

- (b) Six students Ashley (A), Bruce(B), Charles (C), Duane (D), Elke (E), Faith (F) have earn the right to receive a complementary textbook in either algebra (a), calculus (c), differential equation (d), geometry (g), history (h), programming (p), or topology (t). There is only one book on each of these subjects. The preference of the students are
- $A : d, h, t; \quad B : g, p, t; \quad C : a, g, h; \quad D : h, p, t; \quad E : a, c, d; \quad F : c, d, p.$

Can each of the students receive a book he or she likes ?

7.5

- (c) Write the statement of the Kuratowski theorem and show that the graph G is not planar using Kuratowski theorem.

7.5

