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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 3376

Unique Paper Code : 2354000006

Name of the Paper : **Numerical Methods**

Name of the Course : **GE**

Semester : **V**

Programme : **Common Prog Group**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting two parts from each question.

Each question carries equal marks.

Use of non-programmable secientific calculator is allowed.

1. (*a*) Round-off the number 43.96735 correct to four significant digits and then calculate the absolute, relative and percentage errors.

(*b*) Find the relative error and percentage error of the number 6.5, if both of its digits are correct.

(*c*) Determine the number of significant digits in the following numbers :

(I) 438500, (II) 0.0023, (III) 6.56800, (IV) 40.00056, (V) 6.5×10^8.

2. (*a*) Perform five iterations of the Bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 4x + 1 = 0$, also write the order of convergence of Bisection method.

(*b*) Using Regula-Falsi method compute the real root of the equation $x^2 = 3$. Correct up to four decimal places, also write the order of convergence of Regula-Falsi method.

(*c*) Using Newton-Raphson method compute $\sqrt{17}$ correct to four decimal places, also write the order of convergence of Newton-Raphson method.

(*d*) Using Secant method, find the smallest positive root of the equation $x^4 - x = 10$ correct to three decimal digits, also write the order of convergence of Secant method.

3. (*a*) Using Gauss elimination method solve the following system of linear equations :

$$x + 2y + 3z = 1$$

$$2x + 6y + 10z = 0$$

$$3x + 14y + 28z = -8.$$

(*b*) Show that $E - 1 = \frac{1}{2}\delta^2 + \delta\mu$ (Note : Symbols have their own meaning)

(*c*) Find the Lagrange interpolating polynomial which fits into the given data and approximate the value of $f(1.75)$

x	–2	0	2
$f(x)$	4	2	8

4. (*a*) Starting with initial vector $(x, y, z) = (0, 0, 0)$ performs three iterations of Gauss Jacobi method to solve the following system of equations :

$$2x - y + z = -1$$

$$x + 2y - z = 6$$

$$x - y + 2z = -3$$

(*b*) Find the unique polynomial P(x) of degree 2 or less using Newton's interpolating formula for the following data :

x	1	3	4
$f(x)$	1	27	64

Also, estimate P(1.5).

(*c*) Calculate the 3rd divided difference of $\frac{1}{x}$, based on the points x_0; x_1, x_2, x_3.

5. (*a*) Use the formula :

$$f'(x_i) \approx \frac{-f(x_i + 2h) + 4f(x_i + h) - 3f(x_i)}{2h}.$$

To approximate the derivate of $f(x) = \cos x$ at $x_i = \frac{\pi}{2}$, taking $h = 1$, 0.1 and 0.01.

(*b*) For the following data, find f'' (0.04) by using the formula :

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i + h) + f(x_i + 2h)}{h^2}.$$

x	.01	.02	.03	.04	.05	.06
$f(x)$	.1023	.1047	.1071	.1096	.1122	.1148

(c) Calculate by Simpson's rule an approximate value of $\int_{-3}^{3} x^4\, dx$ by taking seven equidistant ordinates.

6. (a) Solve the equation $\frac{dy}{dx} = 1 - y$ with the initial condition $y(0) = 0$ using Euler's method and tabulate the solutions at $x = 0.1$, 0.2 and 0.3.

(b) Using mid-point (Runge–Kutta second order) method, solve the initial value problem $\frac{dy}{dx} = x + \sqrt{y}$ from $x = 0$ to $x = 0.6$ where $y(0) = 1$ by using $h = 0.2$.

(c) Using Runge–Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.