

1614

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8. (a) State and prove Newton-Cote's integration formula.

(b) Solve any **two** of the following :

(i) $u_{x+1} - 2u_x = x^2 \cdot 2^x$

(ii) $u_{x+2} - 2u_x = 2^x$

(iii) $u_{x+2} - 4u_x = 5 \cdot 3^x$ (7,8)

(1000)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1614

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Unique Paper Code : 2372012303

Name of the Paper : DSC-3, Mathematical Analysis

Name of the Course : **B.Sc. Hons. (Statistics) under UGCF-2022**

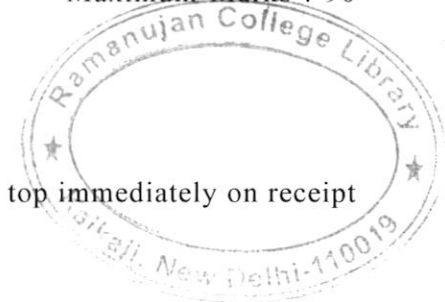
Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Question No. **1** is compulsory.
4. Attempt **five** more questions, selecting **two** from **Section-I** and three from **Section-II**.



P.T.O.

1. Attempt any **five** parts.

(a) Give an example of

(i) A set which is not a neighborhood of any of its points.

(ii) A set which is neither closed nor open.

(iii) A set whose derived set is empty.

(b) Find the limit superior and limit inferior of $\left\langle \frac{1}{2n} \right\rangle$.

(c) Define open set. Let

$$I_n = \left] -n - \frac{1}{n}, n + \frac{1}{n} \right[, n = 1, 2, \dots, \text{Is } \bigcap_n I_n$$

an open set? Justify your answer.

6. (a) Test for convergence the series

$$(i) \frac{1}{\log 2} + \frac{1}{\log 3} + \dots + \frac{1}{\log n} + \dots$$

$$(ii) 1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{2^4} + \dots + \frac{1}{n^n} + \dots$$

(b) Show that the series

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$$

converges absolutely $\forall x$. (8,7)

7. (a) Obtain the power series expression of $\log(1 + x)$.

(b) State Rolle's theorem and give its geometrical interpretation. (9,6)

- (b) Evaluate $(2\Delta + 1)^2(x + 2)^2$, interval of difference being unity. (9,6)

Section-II

5. (a) Show that the series

$$1 + x + x^2 + \dots$$

converges if $0 < x < 1$ and diverges if $x \geq 1$, oscillate finitely if $x = -1$ and oscillate infinitely if $x < -1$.

- (b) Let f be defined on \mathbb{R} by setting (8,7)

$$f(x) = |x - 2| + |x + 2|, \quad \forall x \in \mathbb{R}.$$

Show that f is not derivable at $x = -2$ and $x = 2$.

- (d) (i) Out of total 5 Cote's numbers first two are

$$\frac{7}{90} \text{ and } \frac{32}{90}. \text{ Write the value of other three}$$

Cote's numbers.

- (ii) Show that $\Delta^n f(x) = ah^n(n!)$.

- (e) State Leibnitz test for alternating series. Check

whether $\frac{(-1)^n}{n^3}$ is convergent or not.

- (f) Compute $\Delta(x^3 + 2x^2 + 3x - 1)$.

- (g) Assuming that $n^{1/n} \rightarrow 1$ as $n \rightarrow \infty$, show by applying Cauchy's n^{th} root test that the series

$$\left(\sum_{n=1}^{\infty} n^{1/n} - 1 \right) \text{ converges.} \quad (5 \times 3 = 15)$$

Section-I

2. (a) Let S be a non-empty set of real numbers bounded above. Prove that a real number s is the supremum of S iff the following two conditions hold

(i) $x \leq s, \forall x \in S.$

- (ii) For each positive real numbers ϵ , there is a real number $x \in S$ such that $x > s - \epsilon.$

- (b) What is Cauchy's General Principle of convergence? Show that the sequence defined by

$$\langle a_n = 1 + \frac{1}{4} + \frac{1}{7} + \dots + \frac{1}{3n-2} \rangle, \text{ does not converge.}$$

(8,7)

3. (a) Show that the none of the following sequences $\langle a_n \rangle$ is a Cauchy sequence :

(i) $a_n = (-1)^n$

(ii) $a_n = (-1)^n n$

- (b) Define an open set. Show that the union of two open sets is an open set. (8,7)

4. (a) Show that for the interpolation of $f(x)$ related to $0, \alpha, 1.$ Lagrange's formula gives

$$f(x) = \left[1 - \frac{x(x-\alpha)}{1-\alpha} \right] f(0) + \frac{x(1-x)}{1-\alpha} \times \frac{f(\alpha)-f(0)}{\alpha} + \frac{x(x-\alpha)}{(1-\alpha)} \times f(1)$$

Also show that if $\alpha \rightarrow 0,$ it reduces to

$$f(x) = (1-x^2)f(0) + x(1-x)f'(0) + x^2f(1).$$