8. (a) Using differential under integral sign, evaluate

$$\int_0^a \frac{\log(1+ax)}{1+x^2}\,dx .$$

(b) Find $y_n(0)$, where $y=\left(x+\sqrt{1+x^2}\right)^m$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1633 **G**

Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons) Statistics (NEP-UGCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. **All** questions/parts carry equal marks.

1. (a) If y = $\tan^{-1} x$, then show that

$$y_n=(-1)^{n-1}(n-1)!\sin\left(n\left(\frac{\pi}{2}-y\right)\right)\sin^n\left(\frac{\pi}{2}-y\right)$$

(b) Evaluate $\iint xy(x+y)\,dx\,dy$ over area enclosed by curves $y = x^2$ and $y = x$.

2. (a) Evaluate $\int_0^2 x\left(8-x^3\right)^{1/3} dx$.

(b) Solve

$$x\cos\left(\frac{y}{x}\right)(ydx+xdy) = y\sin\left(\frac{y}{x}\right)(xdy-ydx)$$

3. (a) Evaluate $\lim_{x\to 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$.

(b) Solve $(2D^3 + 3D^2 - 4D - 5)y = \sin x$.

4. (a) Check continuity of function

$$f(x) = \begin{cases} 1-3x, & x < -6, \\ 7, & x = -6, \\ x^3, & -6 < x < 1, \\ 1, & x = 1, \\ 2-x, & x > 1. \end{cases}$$ at $x = -6$ and $x = -1$.

(b) If $x^x y^y z^z$ = constant, then show that

$$\left[\frac{\partial^2 z}{\partial x\,\partial y}\right]_{x=y=z} = -(x\log_e x)^{-1}$$

5. (a) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and evaluate it.

(b) Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

6. (a) Solve $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$.

(b) Prove that $\int_0^1 \frac{\left(1-x^4\right)^{3/4}}{\left(1+x^4\right)^2} dx = \frac{1}{2^{9/2}} \mathbf{B}\left(\frac{7}{4}, \frac{1}{4}\right)$.

7. (a) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos 3y$.

(b) Evaluate $\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$.