

8. (a) Using differential under integral sign, evaluate

$$\int_0^a \frac{\log(1+ax)}{1+x^2} dx.$$

(b) Find  $y_n(0)$ , where  $y = (x + \sqrt{1+x^2})^m$ .

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1633

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Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons) Statistics**  
(NEP-UGCF)

Semester : I

Duration : 3 Hours

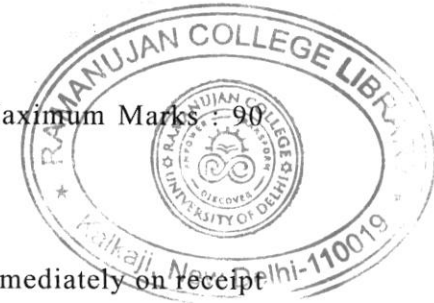
Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. **All** questions/parts carry equal marks.

1. (a) If  $y = \tan^{-1} x$ , then show that

$$y_n = (-1)^{n-1} (n-1)! \sin^n \left( n \left( \frac{\pi}{2} - y \right) \right) \sin^n \left( \frac{\pi}{2} - y \right)$$



(b) Evaluate  $\iint xy(x+y) dx dy$  over area enclosed by curves  $y = x^2$  and  $y = x$ .

2. (a) Evaluate  $\int_0^2 x(8-x^3)^{1/3} dx$ .

(b) Solve

$$x \cos\left(\frac{y}{x}\right) (y dx + x dy) = y \sin\left(\frac{y}{x}\right) (x dy - y dx)$$

3. (a) Evaluate  $\lim_{x \rightarrow 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$ .

(b) Solve  $(2D^3 + 3D^2 - 4D - 5)y = \sin x$ .

4. (a) Check continuity of function

$$f(x) = \begin{cases} 1-3x, & x < -6, \\ 7, & x = -6, \\ x^3, & -6 < x < 1, \text{ at } x = -6 \text{ and } x = -1. \\ 1, & x = 1, \\ 2-x, & x > 1. \end{cases}$$

(b) If  $x^x y^y z^z = \text{constant}$ , then show that

$$\left[ \frac{\partial^2 z}{\partial x \partial y} \right]_{x=y=z} = -(x \log_e x)^{-1}$$

5. (a) Change the order of integration in  $\int_0^1 \int_{x^2}^{2-x} xy dx dy$

and evaluate it.

(b) Solve  $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ .

6. (a) Solve  $(D^3 - 2D + 4)y = x^4 + 3x^2 - 5x + 2$ .

(b) Prove that  $\int_0^1 \frac{(1-x^4)^{3/4}}{(1+x^4)^2} dx = \frac{1}{2^{9/2}} \mathbf{B}\left(\frac{7}{4}, \frac{1}{4}\right)$ .

7. (a) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos 3y$ .

(b) Evaluate  $\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ .