6. (a) Define elementary matrices. Show that elementary matrices are non-singular. Also obtain their inverses.

(b) Using Gram- Schmidt Orthogonalization process, construct an orthonormal basis from

$$x_1 = [2\ 3\ 0],\quad x_2 = [6\ 1\ 0],\quad x_3 = [0\ 2\ 4]$$

7. (a) State Cayley Hamilton theorem. Given $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, express $A^4 - 4A^3 - A^2 + 2A - 5I$ as a linear polynomial in A and hence evaluate it.

(b) Find the characteristic roots and characteristics vectors of the matrix A given by

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

8. (a) Reduce the real quadratic form $x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 + x_3x_1$ to its canonical form and find its rank, signature and index.

(b) Define generalized inverse of a matrix. Give an algorithm for finding a generalized inverse of a $m \times n$ matrix A. Calculate generalized inverse of

$$A = \begin{pmatrix} 2 & 3 & 1 & -1 \\ 5 & 8 & 0 & 1 \\ 1 & 2 & -2 & 3 \end{pmatrix}$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4163 **H**

Unique Paper Code : 2372011203

Name of the Paper : Algebra of Statistics (DSC)

Name of the Course : **B.Sc. (H) Statistics**

Semester : II (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **three** from each Section.
3. **All** questions/parts carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

Section I

1. (a) Define Hermitian and Skew-Hermitian matrices. Show that every square matrix can be uniquely expressed as P + iQ where P and Q are Hermitian matrices.

(b) Solve the system of equations

$$x - 2y + z - w = 0$$
$$x + y - 2z + 3w = 0$$
$$4x + y - 5z + 8w = 0$$
$$5x - 7y + 2z - w = 0$$

2. (a) Find the value of adj(P^{-1}) in terms of P where P is a non-singular matrix and hence show that adj($Q^{-1}B\ P^{-1}$) = PAQ given that adj B = A and $|P| = |Q| = 1$.

(b) Express

$$\Delta = \begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$$

as a product of two determinants and find its value.

3. (a) Define Circulant and Alternant determinants. Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha\beta+\beta\gamma+\gamma\delta)$$

(b) Obtain the inverse of partitioned matrix $\begin{pmatrix} I & P \\ Q & R \end{pmatrix}$ where I is an identity matrix of order s.

4. (a) What are Row-Echelon matrices? Reduce the matrix A to its row echelon form and mark the pivot entries.

$$A = \begin{pmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 6 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{pmatrix}$$

(b) If $F(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $G(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}$

Show that the inverse of matrix $F(\alpha).G(\beta)$ is $G(-\beta).F(-\alpha)$.

Section B

5. (a) Define span of a set and basis. Do the following vectors $a_1 = [1,0,0]$, $a_2 = [0, 1, 0]$, $a_3 = [0, 0, 1]$ form a basis for E^3?

(b) For the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 4 \end{pmatrix}$$

find the non-singular matrices P and Q such that PAQ is in normal form. Hence determine the rank of the matrix A.