$$\frac{a(\bar{X}-\mu_1)+b(\bar{Y}-\mu_2)}{\left[\left\{\frac{(m-1)S_1^2+(n-1)S_2^2}{m+n-2}\right\}\left\{\frac{a^2}{m}+\frac{b^2}{n}\right\}\right]^{\frac{1}{2}}}$$

where a and b are two real fixed nos. and the notations have their usual meanings. (7,8)

6. (a) Prove that if $n_1 = n_2$, then the median of F–distribution is at F = 1 and that the quartiles Q_1 and Q_2 satisfy the condition $Q_1Q_2 = 1$.

(b) Let X_1 and X_2 be independent standard normal random variables. Find the joint distribution of $Y_1 = X_1^2 + X_2^2$ and $Y_2 = X_1/X_2$. Also find the marginal distributions of Y_1 and Y_2. Are Y_1 and Y_2 independent?

(c) State and prove the relation between t and F distributions. (5,5,5)

7. (a) If x is the tabulated value of chi-square distribution with n d. f. at level of f significance α ,then show that $\alpha x \le n$.

(b) If $X \sim F_{n1,n2}$, then show that its mean is independent of n_1.

(c) If the variable t has Student's t-distribution with 2 degrees of freedom, then prove that

$$P(t \ge 2) = \frac{(3-\sqrt{6})}{6}$$ (5,5,5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4354 **G**

Unique Paper Code : 32371301

Name of the Paper : Sampling Distributions

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question no. **1** is compulsory and **two** questions from each Section.

1. Attempt any **five** parts :

(i) If X is a χ^2 variate with n d.f., then prove that for large n, $\sqrt{2X} \sim N(\sqrt{2n}, 1)$.

(ii) Discuss the limiting form of t- distribution .

(iii) Define convergence in distribution and convergence in probability and state the relationship between them.

(iv) If X is an F-variate with 2 and n $(n \ge 2)$d.f. then find the probability P $(X \ge k)$.

(v) A random variable X has the density function e^{-x} for $x \geq 0$. Show that Chebychev's inequality gives $P\{|X - 1| > 2\} < ¼$ and show that the actual probability is e^{-3}.

(vi) Obtain the cumulative distribution function of the first order statistic $X_{(1)}$ in a random sample of size n from a continuous population with cumulative distribution function F(x). Hence find the probability density function of $X_{(1)}$.

(5×3=15)

SECTION I

2. (a) Let $X_1, X_2, ..., X_{2n+1}$ be a random sample from a $N(\mu,\sigma^2)$ population, where n is a positive integer. Find the p.d.f of the sample median and show that it is symmetric about μ. Obtain its expectation.

(b) If X is a random variable and $E(X^2) < \infty$, then prove that $P(|x| \geq a) \geq E(X^2)/a^2$, for all $a > 0$. Use Chebychev's inequality to show that for $n > 36$, the probability that in n throws of a fair die, the number of sixes lies between $\frac{n}{6}-\sqrt{n}$ and $\frac{n}{6}+\sqrt{n}$ is at least 31/36. (7,8)

3. (a) Let $\{X_n\}$ be a sequence of mutually independent random variables such that

$X_n = \pm 1$ with probability $(1-2^{-n}) / 2$ and

$X_n = \pm 2^{-n}$ with probability 2^{-n-}

Examine whether the weak law of large numbers can be applied to the sequence {Xn}.

(b) State and prove central limit theorem. (7,8)

4. (a) Explain the following terms with examples :

(i) null hypothesis and alternative hypothesis

(ii) type I and type II errors,

(iii) critical region,

(iv) p- value.

(b) What is a confidence interval? Obtain the $100(1-\alpha)\%$ confidence limits for the difference of means for large samples. (8,7)

SECTION II

5. (a) Prove that ns^2/σ^2 is distributed as χ^2 distribution with (n-1) d.f., where s^2 and σ^2 are the variances of sample (of size n) and the population respectively.

(b) Let $X_1, X_2, ..., X_m$ and $Y_1, Y_2, ..., Y_n$ be independent random samples from $N(\mu_1,\sigma^2)$ and $N(\mu_2,\sigma^2)$ respectively. If $\bar{X}$ and $\bar{Y}$ denote the corresponding sample means and if $(m - 1)S_1^2 = \sum_{i=1}^{m}(X_i - \bar{X})^2$ and $(n - 1)S_2^2 = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$, obtain the sampling distribution of