(iii) The cheaper the cost per unit in a stratum, the larger should be the size of sample selected from that stratum.

Hence obtain minimum size required for estimating population mean with fixed cost under optimum allocation. (8,7)

8. Write short notes :

(i) Basic Principles of Sample survey

(ii) Non-probability sampling

(iii) Sampling units

(iv) Circular systematic sampling

(v) Sampling errors (3×5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1538 **G**

Unique Paper Code : 2372012301

Name of the Paper : Sample Surveys, DSC – 7

Name of the Course : **B.Sc. (Hons.) STATISTICS (Under NEP-UGCF)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all.

3. **All** questions carry equal marks.

4. Use of non-programmable Scientific Calculator is allowed.

1. (a) What are the main steps involved in conducting a sample survey? Discuss them briefly.

(b) If (X_i, Y_i), i = 1,2,..., N are the pairs of variates defined for every unit of the population of size N and $\bar{x}_n$ and $\bar{y}_n$ are the corresponding means of a simple random sampling without replacement of size n, then prove that :

$$cov(x_i, \bar{y}_n) = \frac{N-n}{Nn} \cdot \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X}_N)(Y_i - \bar{Y}_N) = \frac{N-n}{n(N-1)} cov(X, Y).$$

Also evaluate $E(\bar{x}_n \bar{y}_n)$. (6,9)

2. (a) Discuss briefly about practical difficulties in adopting Neyman's method of allocation of a sample to different strata. How much would the variance increase, on the average, if the allocation is based on the estimates of strata mean squares?

(b) Prove that, in the presence of a linear trend, the variance of a stratified sample is only $\frac{1}{n}$th of the variance of a systematic sample and the latter is also approximately $\frac{1}{n}$th the variance of a random sample. (8,7)

7. (a) Define linear systematic sampling with example. Prove that in systematic sampling positive intra class correlation coefficient between units of same systematic sample inflates the variance of the systematic sample mean. Hence, for maximum value of the intra class correlation coefficient, obtain the efficiency of systematic sampling relative to simple random sampling.

(b) Justify the following statements :–

(i) The smaller the size of stratum, the smaller should be the size of sample to be selected from it.

(ii) The smaller the variability within a stratum, the smaller should be the size of sample selected from the stratum.

Is the ratio method of estimation efficient than sample mean to estimate total yield of barley? Give your answer with all assumptions and mathematical results. Also compute the percentage of gain in precision obtained by estimating mean grain yield using ratio estimator instead of the mean yield of grain per unit if any.

(b) In a population of N units, the number of units possessing a certain characteristic is A and, in simple random sample of size n from it, the number of units possessing the characteristic is a. Obtain an unbiased estimator $\hat{A}$ of A along with its variance $Var(\hat{A})$. Also obtain an estimate of the variance. (8,7)

6. (a) Define simple random sampling without replacement from a finite population. Derive the unbiased estimator of the population mean and find its sampling variance.

(b) A simple random sample of size $n = n_1 + n_2$ with mean $\bar{y}$ is drawn from a finite population of size N by srswor. Further, a simple random sub-sample of size n_1 is drawn from it by srswor with mean $\bar{y}_1$, show that

(i) $V(\bar{y}_1 - \bar{y}_2) = \left(\frac{1}{n_1} + \frac{1}{n_2}\right) S^2$

(ii) $V(\bar{y}_1 - \bar{y}) = \left(\frac{1}{n_1} + \frac{1}{n}\right) S^2$

(iii) $cov(\bar{y}, \bar{y}_1 - \bar{y}_2) = 0$

where, S^2 is the population mean square and $\bar{y}_2$ is the mean of remaining $(n - n_1)$ units.

(8,7)

3. (a) With two strata, a sampler would like to have $n_1 = n_2$ for administrative convenience, instead of using the values given by the Neyman allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given by the $n_1 = n_2$ and the Neyman allocations respectively, show that the fractional increase in variance is

$$\frac{V(\overline{y_{st}}) - V(\overline{y_{st}})opt}{V(\overline{y_{st}})opt} = \left(\frac{r-1}{r+1}\right)^2$$

(b) Show that the relative variance of the regression estimator $\bar{y}_{lr}$ is given by

$$\operatorname{Var}\left(\frac{\bar{y}lr}{\bar{y}N}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)(1-\rho^2)C_y^2$$

to the first approximation (where the notations have usual meaning).

(7,8)

4. (a) Define proportional and Neyman allocations. Compare stratified random sampling under Neyman and proportional allocation with simple random sampling without replacement.

(b) If y and x are unbiased estimators of the population totals of Y and X respectively, show that the variance of ratio estimate $\frac{y}{x}$ can be approximated by $C_y^2 - C_x^2$, where C_x and C_y are coefficients of variation of x and y respectively, (assuming that the correlation coefficient between $\frac{y}{x}$ and x is negligible).

(8,7)

5. (a) In a field of barley, the grain, y_i, and the grain plus straw, x_i, were weighted for each of a large number of sampling units located at random over the field. The total produce (grain plus straw) of the whole field was also weighted. The following data were obtained

C_x	C_y	ρ
1.053	1.063	0.696

where the notations have their usual meaning.