

5. (a) Consider the following payoff matrix of player A and solve it optimally.

| Player A strategies | Player B strategies | | | | |
|---------------------|---------------------|---|----|----|---|
| | 1 | 2 | 3 | 4 | 5 |
| 1 | 3 | 0 | 6 | -1 | 7 |
| 2 | -1 | 5 | -2 | 2 | 1 |

- (b) Obtain an expression for the economic order quantity for an inventory model, when shortages are not allowed. (9,9)
6. Write short notes on the following :
- (a) Phases of Operations Research.
- (b) Two-person zero sum game.
- (c) Unbalanced transportation problem. (6,6,6)

(1000)

[This question paper contains 8 printed pages.]

Your Roll No.....

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Name of the Paper : DSE 1 (a) : Optimization Techniques

Name of the Course : B.Sc. (Hons) Statistics under NEP UGCF

Semester : III

Duration : 3 Hours

Maximum Marks 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **Q1** is compulsory.
4. Use of non-programmable scientific calculator is allowed.



P.T.O.

1. (a) Fill in the blanks :

- (i) While solving L.P. model graphically, the area bounded by the constraints is called _____ .
- (ii) For a maximization LPP, the cost coefficient for an artificial variable is _____ .
- (iii) The assignment problem is a special case of the _____ problem.
- (iv) In a maximization problem of a simplex table, if $Z_j - C_j = 0$ for at least one non-basic variable, then there will be _____ solution.
- (v) A game is said to be strictly determinable if $\max \min \text{ value} = \min \max \text{ value} = \text{_____}$.
- (vi) A _____ is a connected network that may involve only a subset of all the nodes of the network, without having any cycle in between.

(b) Solve the following LP problem using graphical method :

| | Warehouse | | | | | |
|---------|-----------|----|---|----|----|----|
| | A | B | C | D | E | |
| Factory | 1 | 8 | 9 | 12 | 7 | 18 |
| | 2 | 6 | 8 | 13 | 9 | 21 |
| | 3 | 20 | 7 | 10 | 11 | 8 |
| | 4 | 12 | 7 | 14 | 15 | 22 |

- (i) Obtain an initial feasible solution to the distribution problem.
- (ii) What is the optimum distribution for the company?
- (b) Consider the problem of assigning four sales persons to four different sales regions such that the total sales is maximized. The cell entries represent annual sales (in lakhs of rupees).

| Salesman | Sales Region | | | |
|----------|--------------|----|----|----|
| | 1 | 2 | 3 | 4 |
| 1 | 10 | 22 | 12 | 14 |
| 2 | 16 | 18 | 22 | 10 |
| 3 | 24 | 20 | 12 | 18 |
| 4 | 16 | 14 | 24 | 20 |

Find the optimal allocation of sales persons to different regions. (9,9)

way. The map shows the road distances (in miles) directly connecting any two cities. This information is summarized below. A dash indicates that there is no road connecting the respective towns :

| City | Distance (in miles) between adjacent cities | | | | | Destination |
|--------|---|----|----|----|----|-------------|
| | A | B | C | D | E | |
| Origin | 40 | 60 | 50 | - | - | - |
| A | | 10 | - | 70 | - | - |
| B | | | 20 | 55 | 40 | - |
| C | | | | - | 50 | - |
| D | | | | | 10 | 60 |
| E | | | | | | 80 |

Formulate the problem as a shortest route problem where nodes may represent cities, links represent roads and numbers the length of each link in miles. Determine the shortest route for Prachi. (9,9)

4. (a) A company has factories in cities 1, 2, 3 and 4 and supplies to warehouses in cities A, B, C, D and E. Monthly factory capacities are 35, 25, 40 and 32 units respectively. Monthly warehouse requirement are 15, 12, 22, 30 and 20 units respectively. Unit shipping costs are:

$$\text{Maximize } z = 6x_1 + 8x_2$$

Subject to constraints:

$$5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1, x_2 \geq 0$$

- (c) Alpha industry needs 5400 units/year of a bought-out component which will be used in its main product. The ordering cost is Rs. 250 per order and the carrying cost per unit per year is Rs. 30. Find the economic order quantity (EOQ) and the number of orders per year.

- (d) Define a network. Explain the minimal spanning tree problem in networking. (1×6,4,4,4)

2. (a) A firm manufacturing office furniture provides the following information regarding resource consumption, availability and profit contribution. Formulate the problem as a linear programming problem and solve it.

| Resources | Usage per unit | | | Availability |
|------------------------------|----------------|--------|-----------|--------------|
| | Tables | Chairs | Book case | |
| Timber | 8 | 4 | 3 | 640 |
| Assembly Department | 4 | 6 | 2 | 340 |
| Finishing Department | 1 | 1 | 1 | 100 |
| Profit contribution per unit | 30 | 20 | 12 | |

(b) Consider the following L.P.P. :

$$\text{Max. } Z = 5x_1 + 2x_2 + 3x_3$$

subject to constraints

$$x_1 + 5x_2 + 2x_3 \leq a_1$$

$$x_1 - 5x_2 - 6x_3 \leq a_2$$

$$x_1, x_2, x_3 \geq 0$$

where a_1 and a_2 are constants. For specific values of a_1 and a_2 the optimal solution is :

| Basic variable | Solution | x_1 | x_2 | x_3 | x_4 | x_5 |
|----------------|----------|-------|-------|-------|-------|-------|
| x_1 | 30 | 1 | b | 2 | 1 | 0 |
| x_5 | 10 | 0 | c | -8 | -1 | 1 |
| Z-equation | 150 | 0 | a | 7 | d | e |

Evaluate the following :

- (i) The values of a_1 and a_2 that yield the given optimal solution.
- (ii) The values of a, b, c, d and e in the above given optimal solution.
- (iii) The optimal dual solution. (9,9)

3. (a) Solve the following LPP using dual simplex method :

$$\text{Minimize } z = 3x_1 + x_2$$

Subject to constraints:

$$x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

- (b) Prachi is planning a trip by car to another city she has never visited earlier. So, she is studying the road map to locate the shortest route to the destination. She finds that there are five other cities A, B, C, D and E that will come on the