Section C

7. (a) If X and Y are i.i.d exponential variates with parameter λ. Find distribution of $U = \frac{Y}{X}$. Also find expected value of U.

   (b) Define standard laplace distribution. Find its characteristic function and hence find mean and variance. (8,7)

8. (a) Prove that random variable X follows Cauchy distribution with parameters a and b iff random variable X^{-1} follows Cauchy distribution with parameters $\frac{a}{c}$ and $\frac{b}{c}$, where $c = a^2 + b^2$.

   (b) Define logistic distribution. Find its cumulant generating function and hence find mean and variance. (8,7)

9. (a) Let X and Y be i.i.d. standard Cauchy variates. Find the pdf of random variable U = XY.

   (b) Let $X_1 X_2, \ldots, X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, \ldots, X_n)$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ. (8,7)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1576 **G**

Unique Paper Code : 2372012302

Name of the Paper : Advanced Probability Distributions, DSC – 8

Name of the Course : **B.Sc. (Hons.) Statistics (Under NEP – UGCF)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **two** questions from each section.
3. **All** questions carry equal marks.

Section A

1. (a) Let X be the negative binomial variate with parameters k and p. Show that the central moment satisfies recurrence formula :

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\mu_{r-1}\right), r = 1,2, \ldots\ldots$$

   Hence find β_1 and β_2.

(b) Define hyper-geometric distribution and find its mean and variance. (8,7)

2. (a) The trinomial distribution of two r.v.'s X and Y is given by

$$f_{X,Y}(x,y) = \frac{n!}{x!y!(n-x-y)!} p^x q^y (1-p-q)^{n-x-y}$$

For x, y = 0,1,2,..., n and $x + y \leq n$, where $0 \leq p$, $0 \leq q$ and $p + q \leq 1$.

(i) Find the marginal distribution of X and Y.

(ii) Find the conditional distribution of X and Y and hence obtain

(a) $E(X|Y = y)$ (b) $E(Y|X = x)$.

(b) Define 'lack of memory' property. Suppose X is a non-negative integral valued random variable. Show that the distribution of X is geometric if it "lacks memory". (8,7)

3. (a) Obtain Poisson distribution as a limiting case of binomial distribution.

(b) Let X_1, X_2 be independent r.v.'s each having geometric distribution with probability law $p(k) = q^k p$; k = 0,1,2,... Show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform. (8,7)

Section B

4. (a) If X_1 and X_2 are independent rectangular variates on [0,1]. Find the distribution of:

(i) X_1/X_2 (ii) X_1X_2 (iii) $X_1 + X_2$.

(b) Let X and Y be independent gamma variates with parameters μ and ν respectively. Let $U = X + Y$ and $Z = X/(X + Y)$. Find the distributions of random variables. (8,7)

5. (a) If X and Y are two independent random variables with p.d.f. $f(x) = e^{-x}$; $x \geq 0$ and $g(y) = 3e^{-3y}$; $y \geq 0$, then find the p.d.f. of $Z = X/Y$ and also identify the distribution.

(b) Let $X \sim \beta_1(m, n)$ and $Y \sim y(\lambda, m + n)$ be independent random variables, $(m, n, \lambda > 0)$. Find the pdf of XY and identify its distribution. (7,8)

6. (a) Define beta distribution of second kind. Compute mean and variance for beta variate of second kind.

(b) If X is a gamma variate with parameter λ then obtain its m.g.f. Hence deduce that the m.g.f. of standard gamma variate tends to $\exp\left(\frac{t^2}{2}\right)$ as $\lambda \to \infty$. Also interpret the result. (8,7)