Obtain the variance of the one- and two-steps-ahead forecast errors. Further, if $z_N = 1$, $y_N = 4$, $y_{N-1} = 3$, $\sigma_z^2 = 2$, find $\hat{X}_N(2)$ and the standard error of the corresponding forecast error.

(b) For the SARIMA $(0,1,1) \times (1,0,0)_4$ model, obtain the 4-step-ahead forecast at time n. (9,9)

7. Write notes on **any three** of the following :

(a) Effect of detrending a time series,

(b) Moving Average Process,

(c) Dicky-Fuller test,

(d) Selection of trend type. (6,6,6)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4144 **H**

Unique Paper Code : 2372012403

Name of the Paper : Time Series Analysis

Name of the Course : **B.Sc. (Hons.) Statistics, NEP-UGCF**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. Use of simple calculators is allowed.

1. (a) Define a time series. Describe its important components with illustrations.

(b) Explain the additive and multiplicative models of a time series, stating clearly the assumptions in each model. Which of these models is more commonly used and why? (9,9)

2. (a) Give the genesis of the curve

$$X_t = \frac{k}{1+e^{a+bt}}; \quad b<0$$

Hence or otherwise, derive the curve. Explain any one method for fitting the curve.

(b) What is meant by seasonal fluctuations of a time series? How do they differ from cyclic fluctuations? Describe the Link Relative method for measuring the seasonal variations, stating clearly the assumptions made. (9,9)

3. (a) Explain the harmonic analysis method for determining the cyclic component of a time series.

(b) Explain the suitable method for estimating the variance of the random component. (9,9)

4. (a) Describe different types of stochastic time series models. Show that AR(2) process can be written as a weighted moving average of random elements and is infinite.

(b) What is the purpose of Yule-Walker equations? Use Yule-Walker equations to show that for AR(2)

model $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + Z_t$, the values of α_1 and α_2 are $\alpha_1 = \frac{\rho_1(1-\rho_2)}{1-\rho_1^2}$ and $\alpha_2 = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$ where ρ_1 and ρ_2 are autocorrelation functions. (9,9)

5. (a) What is a correlogram of a time series process? "Correlogram is an important tool of analyzing any time series", discuss briefly.

(b) If $X_t = a\cos t\omega$; $t = 1,2,\ldots, N$ where a is a constant and ω is a constant in $(0,\pi]$, then show that $r_k \to \cos k\omega$ as $N \to \infty$. (9,9)

6. (a) For the model

$$(1 - B)(1 - 0.2B)X_t = (1 - 0.5B)Z_t$$

where $\{Z_t\}$ is a discrete-time, purely random process such that $E(Z_t) = 0$, $Var(Z_t) = \sigma_z^2$ and successive values of Z_t are independent, find the forecasts for one- and two- steps-ahead. Hence or otherwise, show that a recursive expression for forecasts three or more steps ahead is given by

$$\hat{X}_N(h) = 1.2\hat{X}_N(h-1) - 0.2\hat{X}_N(h-2)$$