(c) For the autoregressive series $y_{t+2} + \alpha y_{t+1} + \beta y_t = \varepsilon_{t+2}$; $(\alpha^2 - 4\beta > 0)$, show that $1 - r_1 = \dfrac{1+\alpha+\beta}{1+\beta}$ and hence that $1 + \alpha + \beta$ is non-negative. (4,5,6)

6. (a) Describe the Brown's discounted regression procedure for a steady time series model and show that it is equivalent to exponential smoothing.

(b) For the model

$$(1 - B)(1 - 0.2B)y_t = (1 - 0.5B)z_t$$

where $\{Z_t\}$ is a discrete-time, purely random process such that $E(Z_t) = 0$, $Var(Z_t) = \sigma_z^2$ and successive values of Z_t are independent, find the forecasts for one- and two- steps-ahead and compute the variance of their errors. Hence or otherwise, show that a recursive expression for forecasts three or more steps ahead is given by

$$\hat{y}_N(h) = 1.2\,\hat{y}_N(h-1) - 0.2\,\hat{y}_N(h-2)$$

Further, if $z_N = 1$, $y_N = 4$, $y_{N-1} = 3$, $\sigma_z^2 = 2$, find $\hat{y}_N(2)$ and the standard error of the corresponding forecast error. (7,8)

7. Write notes on any **two** of the following :

(a) Selection of trend type

(b) Variate difference method

(c) Bayesian forecasting (7½,7½)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4493 **G**

Unique Paper Code : 32377905

Name of the Paper : Time Series Analysis

Name of the Course : **B.Sc. (Hons.) Statistics – DSE**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** questions carry equal marks.

1. (a) Define a time series. Give any two objectives for analyzing a series.

(b) Define the decomposition of time series by additive and multiplicative hypothesis, clearly outlining the assumptions made.

(c) Identify which component of the time series is mainly responsible for the movement in the following time series :

(i) Decrease in the construction activity during rainy months.

(ii) A need for increased rice production due to increase in population.

(iii) A fall in death rate due to scientific advancement.

(iv) Eruption of a volcano. (5,6,4)

2. (a) Enumerate three properties of the curve

$$U_t = \frac{k}{1+e^{a+bt}}\,;\ b<0$$

and describe its different phases with respect to time series data of annual production of some industry. Justify that the output of an industry follows the logistic trend.

(b) What do you understand by 'seasonal variations' in a time series? Describe the 'Ratio to Trend' method of computing the indices of seasonal variations, stating clearly the assumptions made. (7½,7½)

3. If $\frac{1}{m}[m]$ stands for the simple average of 'm' terms then

(i) prove that

$$\frac{1}{m}[m]y_0\left[y_0 + \frac{m^2-1}{24}\delta^2 y_0\right]$$

(ii) show that

$$\frac{1}{m_1 m_2 \ldots m_r}[m_1][m_2]\ldots[m_r]y_0 = \left[y_0 + \frac{m_1^2+m_2^2+\cdots+m_r^2-r}{24}\delta^2 y_0\right]$$

(iii) if $m_1 = 4$, $m_2 = 4$, and $m_3 = 5$, obtain the weights of the iterated averages when the above formula is approximated by a cubic polynomial. (15)

4. (a) Given that a time series is composed of the trend, the oscillatory, and the random components. Discuss the effect of elimination of the trend on other components of the time series.

(b) Let a time series X_t observed at unit time intervals contain a deterministic sinusoidal component at a known frequency λ,

$$X_t = A_0 + A_1\cos\lambda t + A_2\sin\lambda t + Z_t$$

where Z_t denotes a purely random process. Estimate the unknown parameters A_0, A_1 and A_2. (8,7)

5. (a) Define the terms autocorrelation and correlogram.

(b) Given that z_t is a purely random process such that $E(z_t) = 0$, $V(z_t) = \sigma^2$ and successive values of z_t are independent, for what values of λ_1 and λ_2 is the second order AR process

$$y_t = \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + z_t$$

stationary? Obtain the complementary function of y_t, if $\lambda_1 = 1/3$ and $\lambda_2 = 2/9$.