(b) Let X be a random variable having normal distribution with mean μ and variance σ^2. If $\mu'_r = E(X^r)$, prove that

$$\mu'_{r+2} = 2\mu\mu'_{r+1} + (\sigma^2 - \mu^2)\mu'_r + \sigma^3 \frac{d\mu'_r}{d\sigma}; \qquad r = 0, 1, 2, 3\ldots\ldots$$

Hence, find μ_2 and μ_3. (8,7)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4087 **H**

Unique Paper Code : 2372011201

Name of the Paper : Theory of Probability Distributions

Name of the Course : **B.Sc. (Hons.) Statistics – DSC**

Semester : II (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all selecting three questions from each section.

3. Use of a non-programmable scientific calculator is allowed.

Section – A

1. (a) Define moment generating function $\{M_X(t)]$ of random variable X. Show that

$$\mu'_r = \left[\frac{d^r M_X(t)}{dt^r}\right]_{t=0}; \quad r = 1,2,3...$$

where μ'_r is the r^{th} moment about origin.

(b) Obtain the m.g.f. of the random variable X having p.d.f. :

$$f(x) = \begin{cases} x & ;\text{for } 0 \le x < 1 \\ 2 - x & ;\text{for } 1 \le x < 2 \\ 0 & ;\text{elsewhere} \end{cases}$$

Also, determine mean and variance of X. (8,7)

2. (a) If X is a random variable having cumulants κ_r; r = 1, 2, given by :

$$\kappa_r = (r-1)!\ pa^{-r};\ p > 0,\ a > 0$$

7. (a) Prove the recurrence relation between the moments of Poisson distribution :

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right),\ r = 1, 2, 3,........$$

where μ_r is the r^{th} moment about the mean λ. Hence obtain the skewness and kurtosis of Poisson distribution.

(b) If $X \sim P(\lambda)$, find the m.g.f. for the variable

$$Z = \frac{X-\lambda}{\sqrt{\lambda}}.$$

Find its limiting value when $\lambda \to \infty$. Also interpret the result. (8,7)

8. (a) Define normal distribution and find its mode. Hence show that mode is equal to mean.

$f(x) = e^{-x};\ x \geq 0$ and $g(y) = 3e^{-3y};\ y \geq 0$

Find the joint probability density function of (Z_1, Z_2), where $Z_1 = X/Y$, and $Z_2 = Y$. Also, discuss the independence of Z_1 and Z_2. (8,7)

6. (a) Define uniform distribution for the range (a, b) and find its mean and variance. If X is uniformly distributed with mean 1 and variance $\frac{4}{3}$, find

(i) $P(X < 0)$, and

(ii) $E|X - \bar{X}|$.

(b) Find the mean deviation about mean for $X \sim B(n, p)$. Hence, find mean deviation about mean when $X \sim B\left(5, \frac{1}{2}\right)$. (8,7)

Find (i) Cumulant generating function of A,

(ii) Moment generating function of X, and

(iii) Characteristics function of X.

(b) Let X be a random variable with characteristic function $\phi_X(t)$. Show that

(i) $|\phi_X(t)| \leq 1$, and

(ii) $\phi_{X_1+X_2}(t) = \phi_{X_1}(t) \cdot \phi_{X_2}(t)$;

where X_1 and X_2 are independent random variables. (8,7)

3. (a) Explain the concept of partial correlation coefficient. Show that, in usual notations,

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

(b) Show that $1-R^2_{1.23}=(1-r^2_{12})(1-r^2_{13.2})$.

Deduce that (i) $R_{1.23} \geq r_{12}$

(ii) $R^2_{1.23} = r^2_{12} + r^2_{13}$; if $r_{23}=0$

(8,7)

4. (a) Show that for a trivariate distribution (symbols have their usual meaning)

$$\sigma^2_{1.23}=\sigma^2_1\frac{\omega}{\omega_{11}}$$

(b) Let the joint p.d.f. of random variable X and Y be :

$$f(x,y)=\begin{cases}\frac{1}{16}(x^3+y^3) & ;0<x<2,\ 0<y<2\\ 0 & ;otherwise\end{cases}$$

Find (i) The marginal distribution of X,

(ii) The condition distribution of Y given X = x,

(iii) E(Y|X = x), and

(iv) E(XY|X = $\frac{3}{2}$). (8,7)

Section – B

5. (a) If X_1 and X_2 are independent random variables with common probability function :

$$p(x)=\frac{x}{6};\quad x=1,2,3.$$

Obtain the probability distribution of $Y = X_1 + X_2$.

(b) If X and Y are two independent random variables such that