(ii) Invariance property of consistent estimator

(iii) Optimum properties of ML estimators

(iv) Factorization theorem (4.5,4,4)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2955 **H**

Unique Paper Code : 32371401

Name of the Paper : Statistical Inference

Name of the Course : **B.Sc. (H) Statistics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all, selecting **three** questions from each section.

SECTION I

1. (a) State and prove sufficient conditions for consistency. Are consistent estimators unique? Justify your answer

P.T.O.

(b) Show that MVU estimator is unique.

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from population $f(x,\theta) = (\theta + 1)x^{\theta}, 0 < x < 1, \theta > 1$.

Show that $\left[\frac{-(n-1)}{\sum_{i=1}^{n} \log x_i} - 1\right]$ is an unbiased estimator of θ. (4,4,4.5)

2. (a) State and prove Rao-Blackwell Theorem. Explain its significance in statistical inference.

(b) State Cramer-Rao Inequality. Let $X_1, X_2, \ldots, X_n$ be a random sample from population having p.d.f.

$$f(x,\theta) = \frac{1}{\pi\left[1 + (x-\theta)^2\right]}, \quad -\infty < x < \infty, \ -\infty < \theta < \infty.$$

7. (a) Discuss the method of construction of likelihood ratio test. Consider n Bernoullian trials with probability of success P for each trial. Derive the likelihood ratio test for testing $H_0: P = P_0$ against $H_1 : P > P_0$.

(b) Show that for the distribution: $f(x,\theta) = \theta e^{-\theta x}$; $x > 0, \theta > 0$, central confidence limits for large samples with $(1 - \alpha)$ confidence coefficient is

given by $\theta = \left(1 \pm \frac{Z_{\alpha/2}}{\sqrt{n}}\right) \Big/ \bar{x}$,

where $Z_{\alpha/2}$ is the upper $\frac{\alpha}{2}$ significant point of standard normal distribution. (6,6.5)

8. Write a short note on any **three** of the following :

(i) Method of minimum chi-square

6. (a) Let X has a Poisson distribution with parameter λ. The prior distribution of λ is Gamma distribution

with p.d.f. $g(\lambda)=\frac{e^{-\alpha\lambda}\lambda^{\beta-1}\alpha^{\beta}}{\Gamma(\beta)}$; $x>0, \alpha>0, \beta>0$.

Find the posterior distribution of λ given $X = x$. Also find Bayes' estimator of λ under squared error loss function (SELF).

(b) Define MPCR, UMPCR and UMPUCR. State the theorem used to determine the BCR for testing a simple null hypothesis against the simple alternative hypothesis.

(c) Let $X_1, X_2,..., X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, where μ is known. Obtain UMPCR of size α for testing H_0: $\sigma^2 = \sigma_0^2$ against H_1: $\sigma^2 > \sigma_0^2$. (4,4,4.5)

Find Cramer-Rao Lower bound of variance of an unbiased estimator of θ. (6.5,6)

3. (a) Let $X_1,X_2,...,X_n$ be a random sample from a population having p.d.f. $f(x,\theta)=\frac{e^{-\theta x}\, x^k\, \theta^{k+1}}{\Gamma(k+1)}$; $x>0,\ \theta>0$, where k is a known constant, then obtain the maximum likelihood estimator of θ. Show that the estimator is biased but consistent. Also, show that its asymptotic distribution for large n is $N\left(\theta, \frac{\theta^2}{n(k+1)}\right)$.

(b) Let $X_1, X_2,..., X_n$ be a random sample from Poisson distribution with parameter θ. Using Lehman Scheff's Theorem, find MVUE of

$$\gamma(\theta)=\frac{e^{-\theta}.\theta^{k}}{k!}=P[X=k].$$ (7.5,5)

4. (a) Show that the most general continuous distribution for which the MLE of a parameter θ is the geometric mean of the sample is

$$f(x,\theta)=\left(\frac{x}{\theta}\right)^{\theta\frac{\partial\psi}{\partial\theta}}\exp\left[\psi(\theta)+\xi(x)\right],$$

where $\psi(\theta)$ and $\xi(x)$ are arbitrary functions of θ and x respectively.

(b) Estimate α and β in the case of Pearson's Type III distribution by the method of moments, where

$$f(x,\alpha,\beta)=\frac{e^{\beta x}x^{\alpha\ 1}\beta^{\alpha}}{\Gamma(\alpha)},\ x>0,\ \alpha>0,\ \beta>0.$$ (6.5,6)

SECTION II

5. (a) Given a random sample of size n from a distribution having p.d.f $f(x,\theta)=\frac{1}{\theta}, 0\le x\le\theta$. Show that $100(1-\alpha)\%$ confidence interval for θ is given by

$\left[R,\frac{R}{\psi}\right]$, where ψ is given by $\psi^{n-1}(n-(n-1)\psi)$ $=\alpha$ and R is the sample range.

(b) Let $X_1, X_2,\ldots, X_n$ be a random sample of size n from $N(\mu, \sigma_i^2)$, where σ_i^2 is known. Develop BCR test of size α for testing simple $H_0: \mu=\mu_0$ against a simple $H_1: \mu=\mu_1(>\mu_0)$. In particular, find BCR of size α when $\sigma_i^2=\sigma^2$, $i=1,2,\ldots,n$.

(6,6.5)