7. (a) If $\{X_n\}$ be a sequence of random variables and

$$Y_n = \frac{S_n - E(S_n)}{n}; \; S_n = X_1 + X_2 + \ldots + X_n,$$

then $\lim_{n \to \infty} E\left(\frac{Y_n^2}{1+Y_n^2}\right) \to 0$ is the necessary and sufficient condition for WLLN to hold good.

(b) Let $X_1, X_2, \ldots, X_n$ are independent observations from $N(0,1)$ and $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}} d\theta$. Show that

$$E(\mathrm{Min}(|X_1|, |X_2|, \ldots, |X_n|) = 2^n \int_0^{\infty} (1-\Phi(z))^n \, dz.$$

8. (a) Discuss the test of significance for the difference of means $(\mu_1 - \mu_2)$ of two different populations when large samples from each population are available. Also obtain $100(1 - \alpha)\%$ confidence interval for the difference of means.

(b) Explain the term standard error of sampling distribution. Show that in a sequence of n independent Bernoulli trials with constant probability θ of success, the standard error of proportional of successes is $\sqrt{\frac{\theta(1-\theta)}{n}}$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4068 **H**

Unique Paper Code : 2372012401

Name of the Paper : Sampling Distributions

Name of the Course : **B.Sc. (Hons) Statistics (NEP-UGCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all by selecting **three** questions from each section.
3. **All** questions/parts carry equal marks.

Section – A

1. (a) Derive Brandt and Snedecor formula for computing Chi-square test statistics from observations available in the form of contingency table of size $2 \times k$.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ with sample mean $(\bar{X})$ and sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. Let X_{n+1} be another observation from the same population and independent of $X_1, X_2, \ldots, X_n$. Obtain sampling distribution of $W = \frac{X_{n+1} - \bar{X}}{S}\sqrt{\frac{n}{n+1}}$.

2. (a) For a Chi-Square distribution with n degree of freedom, establish the following recurrence relation between the moments: $\mu_{r+1} = 2r\,(\mu_r + n\,\mu_{r-1})$, $r \geq 1$. Hence, find μ_r; $r = 1,2,3,4$.

(b) Let $X \sim \chi^2_{(n)}$ and if $p_x = P(X \leq x)$ then, show that $x < \frac{n}{1-p_x}$.

3. (a) For t – distribution with n = 4 degree of freedom, show that $P(t \geq 2) = \frac{1}{2} - \frac{5\sqrt{2}}{16}$.

(b) Let X follows t – distribution with n degree of freedom, then, show that $Y = \frac{n}{n + X^2}$ follows Beta distribution.

4. (a) Define F-statistics and derive formula for its probability density function.

(b) If $X \sim F(m,n)$ then, show that

$$Y = \frac{(m/n)X}{1+(m/n)X} \sim B\left(\frac{m}{2}, \frac{n}{2}\right).$$

Deduce variance of X from the variance of Y.

Section – B

5. (a) Let $X_1, X_2, \ldots, X_n$ be a n i.i.d Poisson variates with parameter λ. Use CLT and function $\Phi(x) = (2\pi)^{-\frac{1}{2}}\int_{-\infty}^{x} e^{-\frac{\theta^2}{2}}\, d\theta$ to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \ldots + X_n$; $\lambda = 2$ and $n = 75$.

(b) Show $\lim_{n \to \infty}\left(\int_0^n \frac{e^{-t}t^{n-1}}{(n-1)!}\,dt\right) = \frac{1}{2}$.

6. (a) Let $X_1, X_2, \ldots, X_n$ be n independent observations from exponential distribution with mean μ and having Range $= X_{(n)} - X_{(1)}$. Then, show that

$$E\,(\text{Range}) = \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n-1}\right)\mu.$$

(b) Prove that convergence in mean square implies convergence in probability.