

$\frac{dx}{dt} = 1 + \frac{x}{t}$, $1 \leq t \leq 2$, $x(1) = 1$ taking the step size as

$h = 0.5$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4469

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Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis
(LOCF)

Name of the Course : **B.Sc. (Hons.) Mathematics**

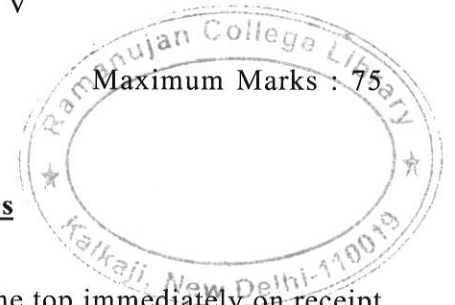
Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of non-programmable scientific calculator is allowed.



1. (a) Discuss the order of convergence of the Newton Raphson method. (6)
- (b) Perform three iterations of the Bisection method in the interval (1, 2) to obtain root of the equation $x^3 - x - 1 = 0$. (6)
- (c) Perform three iterations of the Secant method to obtain a root of the equation $x^2 - 7 = 0$ with initial approximations $x_0 = 2$, $x_1 = 3$. (6)
2. (a) Perform three iterations of False Position method to find the root of the equation $x^3 - 2 = 0$ in the interval (1, 2). (6.5)
- (b) Find a root of the equation $x^3 - 5x + 1 = 0$ correct up to three places of decimal by the Newton's

- (c) Approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 0$ using the first order forward difference formula taking $h = \frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ and then extrapolate from these values using Richardson extrapolation. (6)
6. (a) Using the trapezoidal rule, approximate the value of the integral $\int_3^7 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)
- (b) Derive the Simpson's $1/3^{\text{rd}}$ rule to approximate the integral of a function. (6.5)
- (c) Apply the modified Euler method to approximate the solution of the initial value problem

x	-1	0	1	2
f(x)	3	-1	-3	1

(6)

5. (a) Derive second-order backward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}. \quad (6)$$

- (b) Use the formula

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

to approximate the second derivative of the function $f(x) = e^x$ at $x_0 = 0$, taking $h = 1, 0.1, 0.01$ and 0.001 . What is the order of approximation.

(6)

Raphson method with $x_0 = 0$. In how many iterations does the solution converge? Also write down the order of convergence of the method used. (6.5)

- (c) Explain the secant method to approximate a zero of a function and construct an algorithm to implement this method. (6.5)

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \ 4 \ 1]^T$. (6.5)

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- (b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations. (6.5)

- (c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. (6.5)

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4. (a) Construct the Lagrange form of the interpolating polynomial from the following data :

x	0	1	3
f(x)	1	3	55

(6)

- (b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial.

x	0	1	2	3
y	-1	0	15	80

Hence, estimate the value of $f(1.5)$. (6)

- (c) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data :

P.T.O.