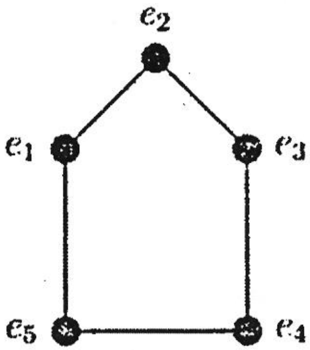
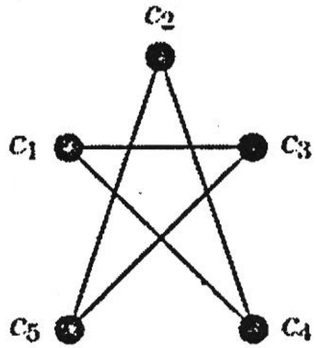
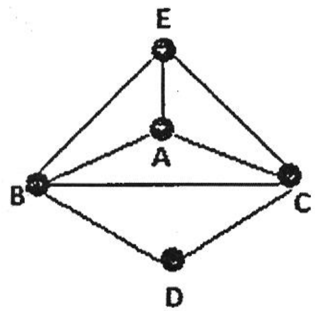



(c) (i) Define a Hamiltonian graph. Is the graph given below Hamiltonian? Explain.




(ii) Show how a Gray Code of length 2 can be constructed using a Hamiltonian cycle.

(5½)



[This question paper contains 8 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 4586** **G**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75



**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the given **eight** questions are compulsory to attempt.

3. Do any **two** parts from each of the given eight questions.

4. Marks for each part are indicated on the right in brackets.

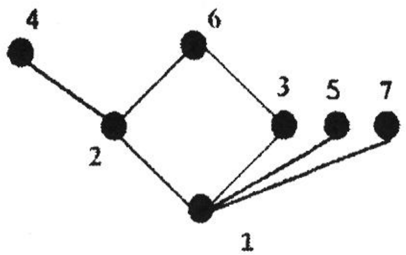
## SECTION I

1. (a) Let P = {a, b, c, d, e, f, u, v}. Draw the Hasse diagram for the partially ordered set (P; ≤), where the relations are given by:

$v < a$, $v < b$, $v < c$, $v < d$, $v < e$, $v < f$, $v < u$,
$a < c$, $a < d$, $a < e$, $a < f$, $a < u$,
$b < c$, $b < d$, $b < e$, $b < f$, $b < u$,
$c < d$, $c < e$, $c < f$, $c < u$,
$d < e$, $d < f$, $d < u$,
$e < u$, $f < u$ (2½)

(b) Give an example of a partially ordered set (P; ≤) which is neither a chain nor an anti chain. Justify with suitable arguments as to why this partially ordered set (P; ≤) is not a chain and not an antichain. (2½)

(c) Consider the diagram below of the ordered subset P = {1, 2, 3, 4, 5, 6, 7} of ($N_0$; ≤), where ($N_0$; ≤) is the ordered set of non-negative integers ordered by relation ≤ on $N_0$ as: For m, n ∈ $N_0$, m ≤ n if m divides n, that is, if there exists k ∈ $N_0$: n=km.

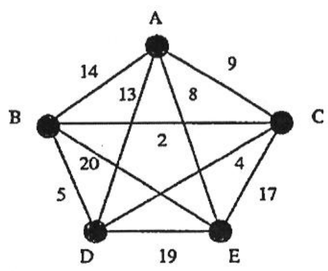





Find the solution, if it exists, for the game of "Instant Insanity" using the above four cubes, where the 6 faces of each of the four cubes have been coloured using four colours red(R), green(G), blue(B) and white(W). (5½)

(c) Explain the Konigsberg bridge problem and formulate it using a corresponding graph. Does the problem have a solution? Give reasons for your answer. (5½)

8. (a) Apply Improved Version of Dijkstra's Algorithm to find shortest distances from vertex A to all other vertices.




(5½)

(b) Consider the two graphs: the pentagon and the star as given below. Compute their adjacency matrices. Are they isomorphic to each other? If yes, exhibit an isomorphism between them. If not, then give suitable argument. (5½)

(b) If B is the set of all positive divisors of 110, then show that (B, gcd, lcm) is a Boolean Algebra. (5)

(c) Find minimal form of the polynomial:

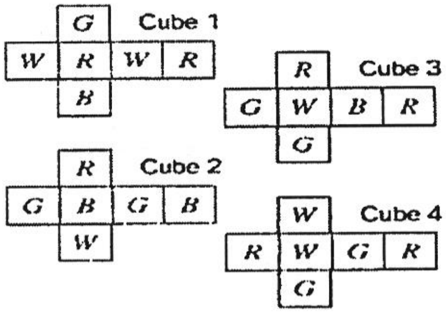
$$f = x'y + x'y'z + xy'z' + xy'z.$$

using Quine's McCluskey method. (5)

## SECTION IV

7. (a) What is the Three houses- Three Utilities Problem? How can it be formulated using graphs? Does this problem have a solution? (5½)

(b) Given four cubes as shown below (the cubes are cut along a few edges, then opened up and flattened) :






For the following subsets of P, find the following meet/join as indicated. Either specify the meet/join if it exists or indicate why it fails to exist.

(i) meet and join of subset {2,3,5}

(ii) meet and join of subset {2,3,6}

(iii) join of P (2½)

2. (a) Show that an order isomorphism for two ordered sets P and Q is a bijection, but the converse is not true. (3)

(b) Let P and Q be ordered sets. Prove that : $(a_1,b_1) \prec (a_2,b_2)$ in P×Q iff ($a_1 = a_2$ and $b_1 \prec b_2$) or ($a_1 \prec a_2$ and $b_1 = b_2$) (3)

(c) Let P, Q and R be ordered sets and let $\varphi: P \to Q$ and $\psi: Q \to R$ be order preserving maps. Then show that the composite map: $\psi o \varphi: P \to R$ given by :
$(\psi o \varphi)(x) = \psi((\varphi(x))$ for $x \in P$, is also an order preserving map. (3)

## SECTION II

3. (a) Let (L, ≤) be a lattice with respect to the order relation ≤. For the operations $\wedge$ and $\vee$ defined on L as:

$$x \wedge y = \inf(x, y),\ x \vee y = \sup(x, y)$$

show that $(L, \wedge, \vee)$ is an algebraic lattice, that is the associative laws, commutative laws, idempotency laws and absorption laws hold. (5)

(b) Define a lattice. Let $D_{24} = \{1,2,3,4,6,8,12,24\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\leq$' defined on $D_{24}$ by $m \leq n$ iff m divides n, then show that $D_{24}$ forms a lattice. (5)

(c) Let $L_1$ and $L_2$ be modular lattices. Prove that the product $L_1 \times L_2$ is a modular lattice. (5)

4. (a) Let L and K be lattices and $f: L \rightarrow K$ be a homomorphism. Then show that the following are equivalent

(i) f is order-preserving

(ii) $(\forall\ a, b \in L)$, $f(a \vee b) \geq f(a) \vee f(b)$

(5½)

(b) Let L be a lattice and let a, b, c Î L. Then show that :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$



(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ (5½)

(c) Define distributive lattice. Prove that homomorphic image of distributive lattice is distributive. (5½)

## SECTION III

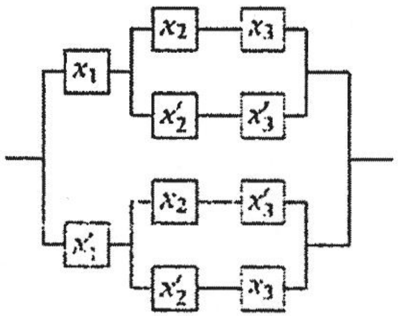
5. (a) Find the disjunctive normal form for (5½)

$$(x_1 + x_2 + x_3)(x_1x_2 + x'_1x_3)'$$

(b) Using Karnaugh diagram, simplify the expression

$$x_3(x_2 + x_4) + x_2x'_4 + x'_2x'_3x_4$$ (5½)

(c) Find symbolic gate representation for (5½)




6. (a) Find the conjunctive normal form of $x_1(x_2+x_3)' + (x/x'_2\ x'_3)x_1$ in three variables. (5)